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## Optical Fiber Technology



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# Generalized study on the pump light propagation in the distributed sidecoupled cladding-pumped fiber with identical pump cores



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### 1. Introduction

High-power fiber lasers have been paid much attention because of their advantages such as compactness, robustness, portability, high efficiency, good beam quality [1–[3\],](#page-6-0) etc. Nowadays, they are playing more and more important roles in various fields such as industry, scientific research and so on. Since 2000, high-power fiber lasers have experienced a rapid development. In 2009 and 2013, the 10-kW singlemode fiber laser [\[4\]](#page-6-1) and 20-kW quasi-single-mode fiber laser [\[5\]](#page-6-2) were reported, respectively.

The development of active fiber such as the double-cladding fiber (DCF) is undoubtedly one most important key driving the rapid development of high-power fiber laser. Recently, one novel side-pumped fiber named as the distributed side-coupled cladding-pumped (DSCCP) fiber (also known as the GT-Wave fiber [6–[9\]](#page-6-3), multi-clad fiber [\[10\]](#page-6-4) or multi-element first cladding fiber [\[11\]](#page-6-5)) becomes more and more attractive. It is distinguished from the DCF by introducing individual pump cores that are physically separated but optically contacted with the inner-cladding of a DCF (named as the signal fiber in the DSCCP fiber [12–[15\]](#page-6-6)). The pump light is injected into each pump core and coupled gradually into the inner-cladding by means of the evanescent wave coupling, and then, pumps the active core in the inner-cladding. Then, the laser can be produced in the active core. Such a unique pumping way makes the DSCCP fiber more beneficial to the pump power scaling and thermal management than the DCF [16–[18\],](#page-6-7) and it is also the reason why the fiber was named as the DSCCP fiber [12–[15,18](#page-6-6)–19].

Recently, a rapid progress has been made on the experimental demonstration of high-power DSCCP fiber lasers and amplifiers. Kilowatt DSCCP fiber lasers with various Master Oscillator Power-Amplifier (MOPA) configurations were demonstrated in 2015 [\[13\]](#page-6-8) and 2016 [\[9\]](#page-6-9). Later, the output power of MOPA DSCCP fiber laser was up-scaled to 8 kW in 2018 [\[20\].](#page-6-10) The experimental study on the DSCCP fiber oscillator was also carried out and kilowatt DSCCP fiber oscillators were also demonstrated [\[21\]](#page-6-11).

In spite of that, the study on the characteristics of DSCCP fiber which is of great importance on fiber designing and optimizing, was not so sufficient. Currently, the pertinent studies were mainly focus on the DSCCP fiber with less-than two pump cores. Refs. [\[19,18\]](#page-6-12) made numerical studies on the pump light propagation and thermal distribution in the DSCCP fiber with one pump core, respectively. Ref. [\[22\]](#page-6-13) numerically studied the pump light propagation in the DSCCP fiber with two pump cores. However, less-than-two pump cores seem not so sufficient for the power-scaling of DSCCP fiber lasers [\[9,20,23,24\].](#page-6-9) Then, how will the DSCCP fiber perform with  $N (N > 2)$  pump cores? It is still a question which needs to be further studied.

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<span id="page-1-1"></span>

Type I

Type II

Type III

Fig. 1. Three basic packing arrangement of  $(N + 1)$  DSCCP fiber: (1) signal fiber, (1a) active core and (1b) inner cladding; (2, 3, ...,  $N + 1$ ) pump cores; (c) common cladding with a lower refractive index.

In this paper, a generalized model describing the pump light propagation in the DSCCP fiber with N identical pump cores (i.e.,  $(N + 1)$ ) DSCCP fiber) is presented, with which the pump coupling and absorption processes corresponding to various pump core numbers are analytically investigated. The paper is arranged as follows. In [Section 2](#page-1-0), the generalized model of DSCCP fiber with identical pump cores is presented. In [Section 3,](#page-2-0) the pump light propagation in the DSCCP fiber is investigated with the model. The variations of pump light coupling and absorption with the pump core number will be discussed in detail. The conclusions will be drawn in [Section 4.](#page-5-0)

#### <span id="page-1-0"></span>2. Generalized model of DSCCP fiber with identical pump cores

Here, we consider three basic arrangements (i.e., Type I, II and III given in [Fig. 1](#page-1-1)) of DSCCP fiber with  $N$  pump cores, which can be assembled to various configurations. As illustrated in [Fig. 1](#page-1-1), each pump cores does not optically contact with each other in Type I arrangement, while each pump cores optically contact with its neighbor ones in Type II arrangement. The Type III arrangement is similar to the Type II arrangement except that the 2nd and  $(N + 1)$ th pump cores do not optically contact with each other.

According to the arrangements of Type I-III, the propagation of pump light between signal fiber and pump cores can be given as follows [\[19,22,25\].](#page-6-12)

Type I arrangement:

$$
\frac{dP_1(z)}{dz} = \sum_{i=2}^{N+1} k_{i1} P_i(z) - \left(\gamma + \sum_{i=2}^{N+1} k_{i1}\right) P_1(z),\tag{1}
$$

$$
\frac{dP_n(z)}{dz} = k_{1n}P_1(z) - k_{n1}P_n(z), \quad (n = 2, 3, \dots, N + 1),
$$
\n(2)

Type II arrangement:

*dP z*

$$
\frac{dP_1(z)}{dz} = \sum_{i=2}^{N+1} k_{i1} P_i(z) - \left(\gamma + \sum_{i=2}^{N+1} k_{1i}\right) P_1(z),\tag{3}
$$

$$
\frac{dP_n(z)}{dz} = k_{1n}P_1(z) + k_{(n-1)n}P_{n-1}(z) + k_{(n+1)n}P_{n+1}(z)
$$

$$
- (k_{n1} + k_{n(n-1)} + k_{n(n+1)})P_n(z) \quad (n = 2, 3, ..., N + 1; n = 2, n - 1 = N + 1; n = N + 1, n + 1 = 2),
$$
(4)

Type III arrangement:

$$
\frac{dP_1(z)}{dz} = \sum_{i=2}^{N+1} k_{i1} P_i(z) - \left(\gamma + \sum_{i=2}^{N+1} k_{i1}\right) P_1(z),
$$
\n(5)  
\n
$$
\frac{dP_n(z)}{dz} = k_{1n} P_1(z) + k_{(n-1)n} P_{n-1}(z) + k_{(n+1)n} P_{n+1}(z)
$$
\n
$$
- (k_{n1} + k_{n(n-1)})
$$
\n
$$
+ k_{n(n+1)}) P_n(z)
$$
\n
$$
\left(n = 2, 3, ..., N + 1; k_{(N+1)2} = 0, k_{2(N+1)} = 0; \atop n = N + 1, n + 1 = 2; n = 2, n - 1 = N + 1 \right),
$$
\n(6)

where N is the number of pump cores, and  $P_1(z)$  and  $P_n(z)$  is the power of the pump light in the inner cladding of the signal fiber and the nth pump cores, respectively.  $k_{ij}$  ( $i = 1, 2, ..., N; j = 1, 2, ..., N; i \neq j$ ) are the coupling coefficients from ith signal fiber (or pump core) to jth signal fiber (or pump core).  $\gamma$  is the absorbing coefficient of the pump light propagating in the inner-cladding of signal fiber (named as the "inner-cladding pump absorption" in the following parts).

Considering that the pump cores are generally designed to be identical in an actual fiber [\[8,9,20,23,24\]](#page-6-14) and thus the coupling coefficients between pump cores should be identical, it is assumed that

$$
\begin{cases}\nk_{12} = k_{13} = ... = k_{1(N+1)} = k_{sp} \\
k_{21} = k_{31} = ... = k_{(N+1)1} = k_{ps} \\
k_{23} = k_{32} = ... = k_{N(N+1)} = k_{(N+1)N} = k_p\n\end{cases} (7)
$$

<span id="page-1-2"></span>Besides, in a practical DSCCP fiber laser, the pump light injected into each pump core (rather than the signal fiber) with approximately identical power [\[8,9,20,23,24\]](#page-6-14), i.e.,

$$
\begin{cases}\nP_1(0) = 0 \\
P_2(0) = P_3(0) = ... = P_{N+1}(0) = P_0\n\end{cases} \tag{8}
$$

Then, it is interesting to find that the pump power transfer in all three arrangements of Type I-III can be expressed with the same numerical model, i.e.,

$$
\frac{dP_1(z)}{dz} = Nk_{ps}P_2(z) - (\gamma + Nk_{sp})P_1(z),\tag{9}
$$

<span id="page-1-3"></span>
$$
\frac{dP_n(z)}{dz} = k_{sp}P_1(z) - k_{ps}P_n(z), (n = 2, 3, ..., N + 1),
$$
\n(10)

where  $P_2(z) = P_3(z) = \ldots = P_{N+1}(z)$ . Thus, we get the generalized model of DSCCP fiber with identical pump cores and injected pump powers. With N equal to 1 and 2, the model can be reduced to the models used in Refs. [\[19,22\]](#page-6-12), respectively. In the following parts, we would like to numerically study the pump power transfer in the  $(N + 1)$ DSCCP fiber with the generalized model.

#### <span id="page-2-0"></span>3. Results and discussions

With the condition given in Eq. [\(8\)](#page-1-2), the distribution of pump power in signal fiber and pump cores can be obtained by solving Eqs. (9)  $\&$ [\(10\),](#page-1-2) i.e.,

$$
P_1(z) = P_0 \frac{N k_{ps}}{\sigma} e^{-\frac{z\alpha}{2}} \left[ e^{\frac{z\sigma}{2}} - e^{-\frac{z\sigma}{2}} \right],\tag{11}
$$

<span id="page-2-7"></span>
$$
P_n(z) = \frac{P_0}{2\sigma} \left[ (\sigma + \beta) e^{-\frac{z(\alpha - \sigma)}{2}} + (\sigma - \beta) e^{-\frac{z(\alpha + \sigma)}{2}} \right] (n = 2, 3, \dots, N + 1),
$$
\n(12)

<span id="page-2-3"></span>where

$$
\alpha = Nk_{sp} + \gamma + k_{ps},\tag{13}
$$

<span id="page-2-6"></span>
$$
\beta = Nk_{sp} + \gamma - k_{ps},\tag{14}
$$

$$
\sigma = \sqrt{\alpha^2 - 4k_{ps}\gamma} \,. \tag{15}
$$

[Fig. 2](#page-2-1) gives the variations of pump power in the signal fiber and pump cores with various N. It can be seen that the normalized pump power (defined as  $[P_n(z)/NP_0]$ ) in each pump core decreases monotonously along the propagation, while the pump power in the signal fiber increases firstly and then decreases when it reaches to a peak value. It can be found that the peak value of normalized pump power in the signal fiber and its corresponding position reduce with the increment of N. Besides, the decrement of normalized pump power also becomes slower with the increment of N. In order to further understand these results, the local absorption coefficient of the pump light will be firstly studied.

#### 3.1. The local absorption coefficient

<span id="page-2-8"></span>With Eqs. (11)  $\&$  (12), the local absorption coefficient can be given as [\[19\]](#page-6-12):

$$
\eta(z) = -\frac{d\left[P_1(z) + P_2(z) + \dots + P_{N+1}(z)\right]}{\left[P_1(z) + P_2(z) + \dots + P_{N+1}(z)\right]dz} = \frac{2\gamma k_{ps}(e^{\sigma z} - 1)}{(\alpha + \sigma)e^{\sigma z} - (\alpha - \sigma)}.
$$
\n(16)

Then, the variation of local absorption coefficient can be calculated

<span id="page-2-1"></span>



<span id="page-2-2"></span>

Fig. 3. The plots of local absorption coefficient via the number of pump cores  $N, N = 1, 3, 5 \ (\gamma = 0.5 \ m^{-1}, k_{sp} = k_{ps} = 1 \ m^{-1}).$ 

and given in [Fig. 3](#page-2-2) (corresponding to the same cases of [Fig. 2](#page-2-1)). It can be found that the local absorption coefficient increases rapidly along the propagation of pump light with  $\eta(0) = 0$  and becomes very close to its up-limitation when the propagation distance is beyond 1 m. Thus, the absorption of pump light in the DSCCP fiber should be mainly determined by the up-limitation of the local absorption coefficient [\[19,22\].](#page-6-12) In spite of that, it is implied that the up-limitation should reduce with the increment of N. This can be understood with the following formula, i.e.,

<span id="page-2-4"></span>
$$
\lim_{z \to \infty} \eta(z) = \frac{2\gamma k_{ps}}{(\alpha + \sigma)}.
$$
\n(17)

Together with the relationship between  $\alpha$  and N given in Eq. [\(13\)](#page-2-3), it can be easily known why the up-limitation decreases with the increment of N which also gives the reason why the decrement of normalized pump power becomes slower with the increment of  $N$  (see [Fig. 2](#page-2-1)). Therefore, the more uniform pump absorption (and thus, the more uniform thermal load) can be expected with the increment of N.

<span id="page-2-5"></span>By substituting Eqs.  $(13)$  &  $(15)$  into Eq.  $(17)$ , we can have that

$$
\frac{\gamma}{\frac{Nk_{sp}}{k_{ps}} + \frac{\gamma}{k_{ps}} + 1} < \lim_{z \to \infty} \eta(z) < \frac{\gamma}{\frac{Nk_{sp}}{k_{ps}} + 1},\tag{18}
$$

which can give some useful results. Firstly, it can be seen that the local absorption coefficient should always be smaller than the absorption coefficient of signal fiber because the terms  $(Nk_{sp}/k_{ps})$  and  $(\gamma/k_{ps})$ should always larger than 0. It also implies that decreasing the values of  $N$ ,  $k_{sp}$  and increasing the value of  $k_{ps}$  are beneficial to improve the pump absorption of DSCCP fiber, which can be illuminated by [Fig. 4.](#page-3-0)

Secondly, it can be found that although the local absorption coefficient is determined by terms  $(Nk_{sp}/k_{ps})$  and  $(\gamma/k_{ps})$  which may have relationship with the areas of pump cores and inner-cladding, it cannot be simply considered proportional to the ratio of the active core area to the total area of pump cores and inner-cladding of signal fiber (similar to the definition of core-to-cladding ratio of DCF, see Ref. [\[18\]\)](#page-6-15). Actually, such a consideration only be approximately effective when every pump core is identical to the inner-cladding of signal fiber (and thus,  $k_{sp} = k_{ps}$ ). The reason is that with this condition, the local absorption coefficient can be roughly estimated as  $[\gamma/(N + 1)]$  (see Eq. [\(17\)](#page-2-4) with  $k_{sp} = k_{ps}$ ) and the inner-cladding area is just the [1/(N + 1)] of the total area (note that  $\gamma$  is the inner-cladding pump absorption of signal fiber).

Thirdly, the local absorption coefficient limitation can be estimated with its maximum value (i.e., the right formula in Eq. [\(18\)](#page-2-5)) if the following condition can be satisfied, i.e.,

<span id="page-3-0"></span>

Fig. 4. The plots of local absorption coefficient via the number of pump cores N, for L11, L21, L31 are the up-limitation of the local absorption coefficient, and L12, L22, L32 are up-limitation of Eq. [\(18\)](#page-2-5). Here, (a) is the comparison of different  $k_{sp}$  ( $\gamma = 1$  m<sup>-1</sup>,  $k_{ps} = 1$  m<sup>-1</sup>), and (b) is the comparison of different  $\gamma$  ( $k_{sp} = 1$  m<sup>-1</sup>,  $k_{ps} = 1 \text{ m}^{-1}$ ).

$$
\frac{Nk_{sp}}{k_{ps}} + 1 > \frac{\gamma}{k_{ps}}.\tag{19}
$$

However, it should be noted that  $\gamma$  should not be too small to ensure large enough pump absorption. Thus, the value of  $(Nk_{sp}/k_{ps})$  should be carefully designed.

Another problem should be addressed is when the local absorption coefficient is close enough to its limitation. In order to solve this problem, we introduce a parameter  $\varepsilon$  ( $\varepsilon$  < 1) defined as the ratio of local absorption coefficient to its up-limitation. Then, with Eqs.  $(16)$  &  $(17)$ , we can also have that when

<span id="page-3-1"></span>
$$
z > \frac{1}{\sigma} \left\{ \ln \left[ 1 - \varepsilon \frac{(\alpha - \sigma)^2}{4\gamma k_{ps}} \right] - \ln(1 - \varepsilon) \right\},\tag{20}
$$

the local absorption coefficient is larger than  $\varepsilon$  times its up-limitation value (see [Appendix A\)](#page-6-16). With Eq. [\(20\),](#page-3-1) the position beyond which the local absorption coefficient can be close to its up-limitation can be obtained, as long as  $\varepsilon$  is close enough to one (the value of  $\varepsilon$  can be given arbitrarily according to the design requirement).

#### 3.2. The length of  $\delta$  total pump absorption

<span id="page-3-2"></span>Besides the local pump absorption, the total pump absorption is another important parameter of DSCCP fiber used as the active fiber. With Eqs.  $(11)$  &  $(12)$ , the total pump absorption can be given as

$$
\delta = 1 - \frac{(P_1(z) + NP_2(z))}{NP_0} = 1 - \frac{\left[ (\alpha + \sigma)e^{\frac{\sigma}{2}z} - (\alpha - \sigma)e^{-\frac{\sigma}{2}z} \right]}{2\sigma}e^{-\frac{\alpha}{2}z}.
$$
\n(21)

When the fiber length is long enough, the second item in the square brackets of Eq. [\(21\)](#page-3-2) is so small compared to the first one that it can be ignored. Thus, Eq. [\(21\)](#page-3-2) can be reduced to

$$
\delta \approx 1 - \frac{(\alpha + \sigma)}{2\sigma} e^{\frac{(\sigma - \alpha)}{2}z}.
$$
\n(22)

<span id="page-3-3"></span>Then, together with Eq. [\(17\)](#page-2-4) the length corresponding to the  $\delta$  total pump absorption can be given as

$$
L_{\delta} = \frac{2\ln\left(\frac{2\sigma(1-\delta)}{\alpha+\sigma}\right)}{\sigma-\alpha} = -\frac{\ln\left((1-\delta)\frac{2\sigma}{\alpha+\sigma}\right)}{\lim_{z\to\infty}\eta(z)} = -\left[\frac{\ln(1-\delta)}{\lim_{z\to\infty}\eta(z)} + \frac{\ln\left(\frac{2\sigma}{\alpha+\sigma}\right)}{\lim_{z\to\infty}\eta(z)}\right].
$$
\n(23)

It can be found that the length is inversely proportional to the uplimitation of local pump absorption. The second term in Eq. [\(23\)](#page-3-3) just gives the effect of the increment process of local pump absorption from 0 to its up-limitation (see [Fig. 3](#page-2-2), the increment process becomes negligible with the propagation distance larger than 1 m). Together with Eqs. [\(13\) & \(15\),](#page-2-7) it can be known that the term  $[2\sigma/(\alpha + \sigma)]$  becomes closer and closer to 1 with the increment of N, which makes the effect of second term weaker. This result can be understood with the help of [Fig. 3](#page-2-2) which shows that the increment process of local pump absorption (and thus its effect) becomes weaker with the larger N. The second term in Eq. [\(23\)](#page-3-3) can be neglected if the following condition can be satisfied, i.e.,

$$
\frac{1}{1-\delta} > \frac{\alpha}{2\sigma} + \frac{1}{2}.\tag{24}
$$

Then, the fiber length can be simply estimated with the values of  $\delta$ and up-limitation of local absorption coefficient  $\eta(z)$  (the first term in Eq. [\(23\)](#page-3-3)). In this case, the active fiber length will increase monotonously with the increment of N because the up-limitation of  $\eta(z)$ decreases monotonously with  $N$  (see Eq.  $(17)$ ). [Fig. 5](#page-4-0) gives some illuminations on the variation of the active fiber length with N. It can be seen that the active fiber length increases monotonously with N. It is also illuminated that the first term in [\(23\)](#page-3-3) can give a well approximation of the fiber length in these cases.

#### 3.3. The peak power in signal fiber

Besides the pump absorption parameters discussed above, the peak pump power in the signal fiber (see [Fig. 2](#page-2-1)) is another factor that should be taken into account, because it can illuminate where the most serious thermal load will be present and how serious the thermal load is. Therefore, we will make some discussions on it in this subsection.

<span id="page-3-4"></span>Firstly, we will reveal where the peak power will be present in the signal fiber. From Eqs.  $(11)$ ,  $(13)$  &  $(15)$  we can have that

<span id="page-4-0"></span>

Fig. 5. The length of  $\delta$  total pump absorption via the number of pump cores N, for  $\delta = 0.9$ , L11, L21, L31 are L<sub> $\delta$ </sub> shown in Eq. [\(23\)](#page-3-3), and L12, L22, L32 are the first term of Eq. [\(23\)](#page-3-3). Here, (a) is the comparison of different  $k_p$  ( $\gamma = 1$  m<sup>-1</sup>),  $k_{ps} = 1$  m<sup>-1</sup>), and (b) is the comparison of different  $\gamma$  ( $k_{sp} = 1$  m<sup>-1</sup>),  $k_{ps} = 1$  m<sup>-1</sup>).

$$
z_{peak} = \frac{1}{\sigma} \ln \left( \frac{\alpha + \sigma}{\alpha - \sigma} \right) = \frac{1}{\sigma} \left( \frac{2\sigma}{\alpha + \sigma} + o \left( \frac{2\sigma}{\alpha + \sigma} \right) \right) \approx \frac{2}{\alpha + \sigma}.
$$
 (25)

Together with the relationship between  $\alpha$ ,  $\sigma$  and  $N$  given in Eqs. [\(13\)](#page-2-7) [& \(15\)](#page-2-7), we can get that  $z_{peak}$  reduces monotonously with the increment of N, which can also be illuminated by [Figs. 2 and 6](#page-2-1). Also from Eqs. [\(13\)](#page-2-7) [& \(15\)](#page-2-7), it can be known that  $z_{peak}$  should reduce monotonously with the  $k_{ps}$ ,  $k_{sp}$  and  $\gamma$  (see [Appendix B\)](#page-6-17). The similar regulations can be drawn from [Fig. 6](#page-4-1), too.

<span id="page-4-2"></span>With Eqs.  $(11)$  &  $(25)$ , the normalized peak pump power in signal fiber can be given as

$$
\frac{P_1(z_{peak})}{NP_0} = \sqrt{\frac{k_{ps}}{\gamma}} \left(\frac{\alpha - \sigma}{\alpha + \sigma}\right)^{\frac{\alpha}{2\sigma}} = \sqrt{\frac{k_{ps}}{\gamma}} \left(1 - \frac{\alpha}{\alpha + \sigma} + o\left(\frac{\alpha}{\alpha + \sigma}\right)\right).
$$
(26)

Again, with the help of Eqs.  $(13)$  &  $(15)$ , it can be known that the ratio should decrease monotonously with the increment of  $N$ , which means that with a given total pump power (i.e.,  $NP_0$ ), the peak pump power coupled into the signal fiber should decrease with the increment of N. Then, it means that the thermal load induced by the peak pump power will be lowered with the increment of N if  $\gamma$  is unvaried. Besides, Eq. [\(26\)](#page-4-2) also implies that the ratio should reduce monotonously with the  $k_{sn}$ and  $\gamma$  and increase with  $k_{ps}$ . Thus, the ratio can be elevated by decreasing the terms ( $k_{sp}/k_{ps}$ ) and ( $\gamma/k_{ps}$ ). These dependences are also illuminated in [Fig. 7.](#page-5-1)

<span id="page-4-3"></span>In spite of that, the peak pump power in the signal fiber should be given as

$$
P_1(z_{peak}) = NP_0 \sqrt{\frac{k_{ps}}{\gamma}} \left( \frac{\sqrt{4k_{ps}\gamma}}{\alpha + \sigma} \right)^{\frac{\alpha}{\sigma}} = NP_0 \sqrt{\frac{k_{ps}}{\gamma}} \left( 1 - \frac{\alpha}{\alpha + \sigma} + o\left(\frac{\alpha}{\alpha + \sigma}\right) \right).
$$
\n(27)

It is interesting to find that with a given pump power injected into each pump core (i.e.,  $P_0$ ), the peak pump power will increase monotonously with the increment of  $N$  (also see [Fig. 8](#page-5-2)). It means that although the

<span id="page-4-1"></span>

**Fig. 6.** The position of the peak pump power in signal fiber via the number of pump cores N. Here, (a) is the comparison of different  $k_{sp}$  ( $\gamma = 1$  m<sup>-1</sup>,  $k_{ps} = 1$  m<sup>-1</sup>), and (b) is the comparison of different  $\gamma$  ( $k_{sp} = 1 \text{ m}^{-1}$ ,  $k_{ps} = 1 \text{ m}^{-1}$ ).

<span id="page-5-1"></span>

**Fig. 7.** The normalized peak pump power of signal fiber via the number of pump cores N. Here, (a) is the comparison of different  $k_{sp}$  ( $\gamma = 1$  m<sup>-1</sup>,  $k_{ps} = 1$  m<sup>-1</sup>), and (b) is the comparison of different  $\gamma$  ( $k_{sp} = 1 \text{ m}^{-1}$ ,  $k_{ps} = 1 \text{ m}^{-1}$ ).

normalized peak pump power coupled into the signal fiber will decrease with the increment of  $N$ , the peak pump power will increase (and then, the pertinent thermal load should increase, correspondingly) because of the increment of total pump power with  $N$ . This result implies that the peak pump power and pertinent thermal load in the signal fiber should not be lowered with the increment of N with a given  $P_0$  (general the practical case where the total pump power can be increased by increasing N). This result also gives the advantage of distributed-pumping scheme of Refs. [\[13,21\]](#page-6-8) in the thermal load suppression, because it does not increase the total pump power simply by increasing  $N$  but by increasing the number of gain stage with the DSCCP fiber of small  $N$  (and then, the total pump power can be increased with the thermal load in each gain stage not increased). Besides, from Eq. [\(27\)](#page-4-3), we can also have that

$$
\lim_{n \to \infty} P_1(z_{peak}) = P_0 \frac{k_{ps}}{k_{sp}},\tag{28}
$$

which gives the up-limitation of variation of pump power in signal fiber at  $z_{peak}$  with N.

Finally, we would like to make some discussion on the local pump absorption at the position of  $z_{peak}$ , i.e.,

$$
\eta(z_{peak}) = \frac{\gamma k_{ps}}{\alpha} = \left(\frac{1}{2} + \frac{\sigma}{2\alpha}\right) \lim_{z \to \infty} \eta(z). \tag{29}
$$

It means that the position of  $z_{peak}$  should be just the position where the local absorption coefficient is  $\varepsilon$  times its up-limitation (i.e.,  $\varepsilon$  is equal to  $[1/2(1 + \sigma/\alpha)]$  in Eq. [\(20\)\)](#page-3-1). Then, the  $z_{peak}$  can be used as the position threshold beyond which the local pump absorption is approximately equal to its up-limitation, as long as the ratio  $\sigma/\alpha$  is close enough to one.

## <span id="page-5-0"></span>4. Conclusion

In this paper, a generalized numerical model describing the pump

<span id="page-5-2"></span>

**Fig. 8.** Peak pump power in signal fiber via the number of pump cores N. Here, (a) is the comparison of different  $k_{sp}$  ( $\gamma = 1$  m<sup>-1</sup>,  $k_{ps} = 1$  m<sup>-1</sup>), and (b) is the comparison of different  $\gamma$  ( $k_{sp} = 1 \text{ m}^{-1}$ ,  $k_{ps} = 1 \text{ m}^{-1}$ ).

light propagation in the DSCCP fiber with  $N$  identical pump cores is presented, and the pump light propagation in the DSCCP fiber is analytically studied. It is found that the local pump absorption should be always smaller than the inner-cladding pump absorption of signal fiber, and it can be increased by decreasing the values of  $(Nk_{sp}/k_{ps})$  and  $(\gamma/$  $k_{ps}$ ). It is also revealed that the length of DSCCP fiber for  $\delta$  total pump absorption is inversely proportional to the up-limitation of local pump absorption, and thus, increasing monotonously with the increment of N. The peak pump power in the signal fiber and its position are also discussed. It is found that although the normalized peak pump power decreases monotonously, the peak pump power increases monotonously with the increment of  $N$ . This result implies that the value  $N$ and  $P_0$  should be carefully designed in order to optimize the thermal

load produced in the signal fiber. It also gives the thermal management advantage of the pumping scheme with multiple gain stages presented in Refs. [\[13,21\]](#page-6-8). It is also found that there is an up-limitation of peak pump power with the increment of N, which is determined by the ratio of  $k_{ps}$  and  $k_{sp}$ . Besides, the dependence of the pump light propagation on  $k_{ps}$ ,  $k_{sp}$  and  $\gamma$  are also given analytically. We believe these results can provide significant guidance on understanding and designing of DSCCP fiber and pertinent lasers and amplifiers.

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## <span id="page-6-16"></span>Appendix A.  $\varepsilon$  times of up-limitation of the local absorption coefficient

<span id="page-6-18"></span>If the local absorption coefficient reach  $\varepsilon$  times its up-limitation value, and from Eqs. [\(16\) and \(17\)](#page-2-8), we can get

$$
\frac{(\alpha - \sigma)}{2} \varepsilon = \frac{2\gamma k_{ps}(e^{\sigma z} - 1)}{(\alpha + \sigma)e^{\sigma z} - (\alpha - \sigma)}.
$$
\n(A1)

<span id="page-6-19"></span>So Eq. [\(A1\)](#page-6-18) can change to

$$
\frac{(\alpha - \sigma)}{2} \varepsilon [(\alpha + \sigma) e^{\sigma z} - (\alpha - \sigma)] = 2\gamma k_{ps} (e^{\sigma z} - 1).
$$
\n(A2)

<span id="page-6-20"></span>By substituting Eqs.  $(13)$  &  $(15)$  into Eq.  $(A2)$ , we can have that

$$
2\gamma k_{ps} - \varepsilon \frac{(\alpha - \sigma)^2}{2} = 2\gamma k_{ps} e^{\sigma z} (1 - \varepsilon).
$$
 (A3)

When the local absorption coefficient is larger than  $\epsilon$  times its up-limitation value, we can get from formula [\(A3\)](#page-6-20) that the position *z* satisfy

$$
z > \frac{1}{\sigma} \left\{ \ln \left[ 1 - \varepsilon \frac{(\alpha - \sigma)^2}{4\gamma k_{ps}} \right] - \ln(1 - \varepsilon) \right\},\tag{A4}
$$

which is equal to Eq. [\(20\).](#page-3-1)

### <span id="page-6-17"></span>Appendix B. The peak pump power in signal fiber

By substituting Eq.  $(15)$  into Eq.  $(25)$ , we can have that

$$
\frac{2}{\alpha + \sqrt{\alpha^2 - 4k_{ps}\gamma}} = \frac{1}{\alpha \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{k_{ps}\gamma}{(k_{sp} + k_{ps} + \gamma)^2}} \right)}.
$$
(B1)

We can see the term [ $k_{ps}\gamma/(k_{sp}+k_{ps}+\gamma)^2$ ] decreases with the increase of  $k_{ps}$ ,  $k_{sp}$  and  $\gamma$ . Together with the Eq. [\(13\),](#page-2-3) we can see  $z_{peak}$  should reduce monotonously with the  $k_{ps}$ ,  $k_{sp}$  and  $\gamma$ .

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