Modulation instability of surface plasmon polaritons in graphene double-layer structure

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ABSTRACT

A physical model of compact generator of infrared surface plasmon polaritons based on a planar waveguide structure to produce short pulses with a controllable repetition rate is proposed. The pulse generation is produced by modulation instability of continuous surface plasmon polariton waves in a film structure with graphene sheets (two graphene sheets spatially separated by a dielectric layer).

Keywords: graphene, surface plasmon polaritons, nonlinear optics, induced modulation instability

1. INTRODUCTION

Modulation instability (MI), a fundamental effect provoking an explosive growth of weak perturbations in a nonlinear physical system [1], underlies the operation of the high repetition rate optical pulse generator. In optics, the MI can be employed for transformation of continuous weakly modulated wave into a pulse train. Propagation of the modulated wave in an optical waveguide [2] is one of the most illustrative cases of MI developed in the systems described by the nonlinear Schrödinger equation. The similar processes could be observed for the surface plasmon polaritons (SPPs) in plasmonic structures [3-6]. The nonlinear response can be extremely enhanced if a graphene-dielectric heterostucture is used [7-9]. In this paper, we propose a physical model of compact generator of infrared surface waves based on graphene double-layer structure to produce short SPP pulses. The pulse generation is produced by MI of SPPs in two graphene sheets spatially separated by a GaAs layer with a thickness of the order of tens of nanometers. The initial conditions correspond to a continuous SPP wave with the intensity perturbed in space with small modulation. For the layer thickness of 25 nm, it is shown that during a time interval less than 20 nanoseconds the modulated surface wave is transformed into a train of pulses due to nonlinear effects.

2. MAIN EQUATIONS

Consider a double-layer graphene structure consisting of a dielectric of permittivity ε_2 filling the space between two graphene sheets of spacing d and a dielectric of permittivity ε_1 filling the rest of the space (Fig. 1). Both graphene sheets have the same properties characterized by a scattering rate and a Fermi energy. The magnetic susceptibilities of media in the optical range are assumed to be unity. The z-axis be oriented perpendicular to graphene sheets, and the direction of surface wave propagation be the x-axis.

The characteristic equation to be solved for the SPP dispersion of double-layer graphene structure is [10,11]

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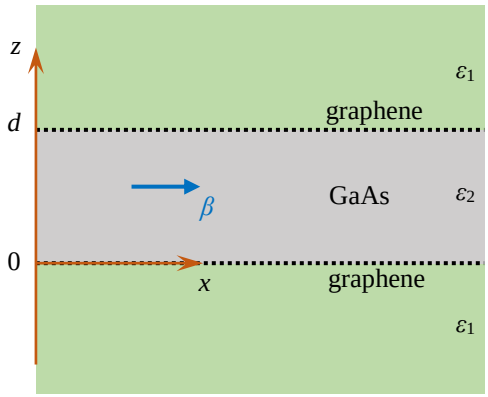

Figure 1. Structure with two graphene sheets separated by a GaAs spacer of thickness d and relative permittivity ε_2. The graphene/GaAs/graphene structure immersed in dielectric media with permittivity ε_1.

$$\left[\left(\frac{\varepsilon_1}{q_1}+\frac{i\sigma}{\omega\varepsilon_0}\right)\cosh\left(q_2\frac{d}{2}\right)+\frac{\varepsilon_2}{q_2}\sinh\left(q_2\frac{d}{2}\right)\right]\left[\left(\frac{\varepsilon_1}{q_1}+\frac{i\sigma}{\omega\varepsilon_0}\right)\sinh\left(q_2\frac{d}{2}\right)+\frac{\varepsilon_2}{q_2}\cosh\left(q_2\frac{d}{2}\right)\right]=0, \quad (1)$$

where $q_j=\sqrt{\beta^2-k_0^2\varepsilon_j}$ $(j=1,2)$ is the attenuation coefficient, i.e. z-component of the imaginary wave vectors in the film and dielectrics, $k_0=\omega/c$, c is the speed of light in vacuum, β is the propagation constant, i.e. x-component of the SPP wave vector. The value $\mathrm{Re}[q_j]\geq 0$ determines the rate of SPP amplitude decreasing with increasing distance from the interfaces (graphene sheets) of the structure. The first product term in Eq. (1) corresponds to the symmetric mode with the electric field z dependence even with respect to $z = d/2$ [$E_x(x,y,0)=E_x(x,y,d)$], while the second term describes the antisymmetric mode, odd with respect to the central plane [$E_x(x,y,0)=-E_x(x,y,d)$].

Let us consider a continuous surface wave with the carrier frequency ω_0 and corresponding propagation constant β_0. A standard procedure is applied to define the ratio between the dispersion and nonlinear effects. It involves expansion of the function $\omega=\omega(\beta,I)$ in a Taylor series about a point ($\beta=\beta_0$, $I=0$) [12, 13]:

$$\omega\approx\omega_0+(\beta-\beta_0)\left.\frac{\partial\omega}{\partial\beta}\right|_{\beta_0}+\frac{1}{2}(\beta-\beta_0)^2\left.\frac{\partial^2\omega}{\partial\beta^2}\right|_{\beta_0}+I\left.\frac{\partial\omega}{\partial I}\right|_0, \quad (2)$$

where the last term indicates that the nonlinearity of a structure causes the frequency dependence on the wave intensity I. Applying

$$\Omega=\omega-\omega_0,\ K=\beta-\beta_0,$$

Eq. (2) can be reduced to the nonlinear dispersion ratio between the frequency and propagation constant:

$$\Omega=\upsilon_g K+\frac{1}{2}DK^2+\gamma I, \quad (3)$$

where

$$\upsilon_g=\left.\frac{\partial\omega}{\partial\beta}\right|_{\beta_0} \quad (4)$$

is the group velocity,

$$D = \left.\frac{\partial^2 \omega}{\partial \beta^2}\right|_{\beta_0} \tag{5}$$

is the parameter determined by the group velocity dispersion,

$$\gamma = \left.\frac{\partial \omega}{\partial I}\right|_0 = \left.\frac{\partial \beta}{\partial I}\right|_0 \upsilon_g \tag{6}$$

is the coefficient of Kerr nonlinearity at the frequency $\omega = \omega_0$. Note that the values of the derivatives in Eqs. (4) - (6) are taken for $I = 0$.

Eq. (3) can be rewritten using the formalism of slowly varying amplitudes $A(x,t)$. The wave field is described as $A(x,t)\exp\left[i(\beta_0 x - \omega_0 t)\right]$, where the oscillation periods in time and the x coordinate of the envelope $A(x,t)$ are $\Omega << \omega_0$ and $K << \beta_0$. Using the relations

$$\frac{\partial A}{\partial t} = -i\Omega A, \quad \frac{\partial A}{\partial x} = iKA \ ,$$

the dispersion relation (3) could be linked to nonlinear differential equation that is the nonlinear Schrödinger equation describing the dynamics of the SPP complex amplitude:

$$\frac{\partial A}{\partial t} + \upsilon_g \frac{\partial A}{\partial x} - \frac{i}{2} D \frac{\partial^2 A}{\partial x^2} + i\gamma |A|^2 A = 0 \ , \tag{7}$$

where $|A|^2 = I$ is the intensity. In the running coordinate system $X = x - \upsilon_g t$ Eq. (7) can be rewritten as

$$\frac{\partial A}{\partial t} - \frac{i}{2} D \frac{\partial^2 A}{\partial X^2} + i\gamma |A|^2 A = 0 \ . \tag{8}$$

The particular solution of Eq. (8)

$$A_0 = \sqrt{I}\exp(-i\gamma I t) \tag{9}$$

describes SPP with the amplitude constant along the x coordinate ($\partial A_0 / \partial x = 0$) and nonlinear shift γI of the carrier frequency.

We assume that the field of stationary wave excited in the layer is perturbed by a small harmonic wave δ with the wavenumber K and frequency Ω satisfying Eq. (3), and the amplitude varying with time according to the factor $\exp\left[Gt\right]$. Stability analysis for the perturbed solutions

$$A = A_0 + \delta(\Omega, K)\exp\left[Gt - i\gamma I t\right] = \left(\sqrt{I} + \delta(\Omega, K)\exp\left[Gt\right]\right)\exp\left[-i\gamma I t\right]$$

shows that the MI regime is established if the dispersion D and nonlinearity γ are of opposite signs [14].

In MI regime ($D\gamma < 0$, $G > 0$), the harmonic perturbations δ at the wavenumbers $0 < |K| < |K_0|$ grow in time with the increment

$$G = |D| K^2 \sqrt{4I \frac{|\gamma|}{|D| K^2} - 1} \ . \tag{10}$$

The range of MI regime is limited by $|K_0| = 2\sqrt{I|\gamma / D|}$. The value of the maximal increment $G_{\max} = 2|\gamma| I$ is achieved for the wavenumber

$$\left|K_{\max}\right| = \sqrt{2I\left|\frac{\gamma}{D}\right|} = \frac{1}{\sqrt{2}}\left|K_0\right|. \tag{11}$$

3. MATERIAL PARAMETERS

According to random-phase approximation [15] and the Kubo formula [16], we use simplified version of surface conductivity as [17]

$$\sigma = \sigma_L + \sigma_{NL}\left|E_\tau\right|^2 \tag{12}$$

in which linear and nonlinear terms are

$$\sigma_L = i\frac{eE_F{}^2}{\pi\hbar^2\omega}, \tag{13}$$

$$\sigma_{NL} = -i\frac{3}{8}\frac{e^4}{\pi\hbar^2}\frac{V_F^2}{E_F\omega^3}. \tag{14}$$

Here, subscript τ refers to the field component tangential to the interface, E_F is the Fermi energy, $V_F \approx c/300$ is the Fermi velocity.

The film is assumed to be made of GaAs. Hereinafter, we consider a symmetric structure, in which the distribution of the dielectric permittivity is described by a step function

$$\varepsilon(z) = \begin{cases} \varepsilon_1 = 1, & z < 0, z > d, \\ \varepsilon_2 = 10.5, & 0 < z < d. \end{cases} \tag{15}$$

4. RESULTS OF NUMERICAL SIMULATION

Formulaic expressions presented in section 2 are obtained in approximation of the undepleted pump field I, and, thus, are valid only in the early stages of the development of MI. Numerical calculations taking into account the power decreasing of the initial continuous wave (pump wave) show that this process is described by breather solution [2], i.e. the formed pulse sequence is transformed further into the modulated wave that, in its turn, decomposes into a train of pulses again, and so on.

Fig. 2 shows the results of numerical simulations of Eq. (8) for GaAs spacer of thickness 25 nm. We consider the SPP mode with negative nonlinear coefficient and positive dispersion parameter. For this mode, the condition $D\gamma < 0$ of MI regime is fulfilled for used parameters. The initial conditions correspond to continuous SPP wave with the intensity I perturbed in space with small modulation: $A(t=0) = \sqrt{I}\left(1 + 0.02\cos\left(2\pi x/\Delta X\right)\right)$. The parameters of SPPs in the double-layer graphene structure calculated from Eqs. (1), (4)-(6) for the intensity of $I = 10^{-2}$ W/μm^2 are: the dispersion $D \approx 3.5\cdot10^{-5}$ m^2/s, the nonlinear coefficient $\gamma \approx -3.55\cdot10^{-3}$ m^2/(W×s). The spatial period of modulation is about 1.4 μm. The numerical estimations show that the SPPs are induced in the structure by optical radiation at $\lambda \approx 6.8$ μm. During time interval of 20 nanoseconds the modulated wave is transformed into a train of pulses. The duration of each SPP pulse is $\tau_m < 10$ ns, time interval between pulses is less than 30 nanosecond, and the peak intensity is in 6 times higher than the intensity of the original SPP wave.

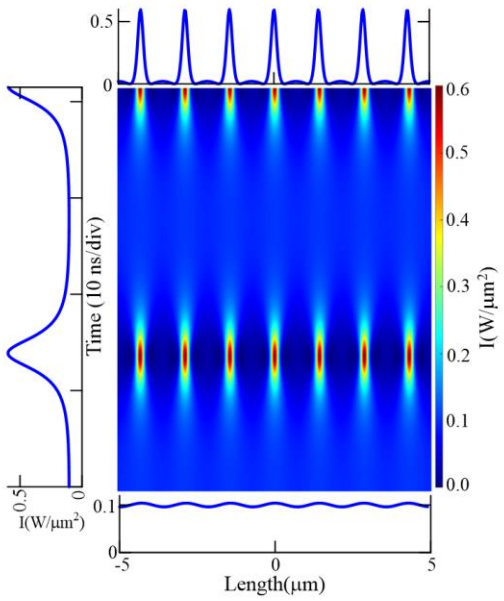

Figure 2. Evolution of the SPP intensity in the film of thickness 25 nm; the initial wave (at $t = 0$), modulated along x (in bottom), time dynamics of the wave intensity at $x = 0$ (left), spatial distribution of the SPP intensity (top) at $t = 41.4$ ns.

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