ISSN 0031-918X, Physics of Metals and Metallography, 2019, Vol. 120, No. 3, pp. 238–246. © Pleiades Publishing, Ltd., 2019.
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ELECTRICAL AND MAGNETIC PROPERTIES

Response of Magnetic Nanoparticles Lattice to Gaussian Pulse of Magnetic Field

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Received July 10, 2018; revised August 14, 2018; accepted October 15, 2018

Abstarct—The response of uniform and binary plane lattices of magnetic nanoparticles with uniaxial anisotropy to a short Gaussian pulse of magnetic field is studied in this work. It is shown that the dipole–dipole interaction and the presence of two types of nanoparticles modulate and decrease the response amplitude. The complicated periodic dependence of the amplitude and duration of the system response on the duration and peak value of the applied pulse has been discovered. The possibility of controlling the magnetization reversal of both the entire lattice and its parts by choosing the lattice composition and parameters of the bias field and pulse is demonstrated.

Keywords: magnetic moment of lattice of dipoles, uniaxial anisotropy, magnetization reversal of the system by pulse, response of the system to magnetic-field pulse, uniform and binary lattices

DOI: 10.1134/S0031918X1903013X

INTRODUCTION

Nanotechnological magnetic structures and arrays of magnetic nanoparticles have been intensively studied [1–7] recently. Among these structures, two-dimensional magnetically-ordered structures are considered to be the most interesting, e.g., they can be composed of single-domain nanoparticles of ferromagnetic alloys [8]. The dipole–dipole interaction is the main interaction of magnetic moments in these lattices [8, 9]. In [10–13], equilibrium states and dynamic modes of the magnetization reversal of linear chains and square lattices of nanoparticles in external magnetic field were considered. The magnetization reversal of lattices of magnetic nanoparticles with a cubic crystalline anisotropy, which are coupled by the dipole–dipole interaction, was studied in [14]. It has been shown that the discrete nature of these structures causes substantial differences between the equilibrium states and dynamic modes of the magnetization reversal of such chains and lattices and the properties of macroscopic single-domain objects. In particular, these differences include bistable states which are caused by the existence of several orientation configurations differing in the total magnetic moment of the system, as well as possible transitions between these configurations and dynamic oscillation modes of the magnetic moment of the system upon the magnetization reversal.

In addition, ordered arrays of magnetic nanoparticles of various dimensions can be used as a medium for ultra-dense recording and the storage of information. The information recording onto the lattice of magnetic dipoles is based on the change in the equilibrium configuration of magnetic moments by radio pulses of magnetic field; the reading is done by excitation of the arisen configuration by a low-power radio pulse with a frequency corresponding to the ferromagnetic resonance and by detection of the response of the dipole system [15–17].

In this work, the response of the 6×6 plane lattice of magnetic nanoparticles to a magnetic-field pulse is studied by the numerical calculation of dynamics equations. Since the uniaxial anisotropy is the most observed and important for information recording, contrary to [14], the uniaxial nanoparticles are considered. Together with lattices consisting of only one type of nanoparticles (uniform lattices), lattices of two alternating types of nanoparticles, which differ in value of magnetic moment, (binary) are investigated. An effect of parameters of the Gaussian pulse on the amplitude of oscillations of magnetic moments and the duration of the response, as well as the magnetization reversal of the uniform and binary lattices under the action of pulse magnetic field are studied.

INITIAL EQUATIONS

Let us consider the two-dimensional 6 × 6 array of nanoparticles with the magnetic moment $|\mathbf{m}_i| = m_i$ and a shape close to spherical. We assume that each nanoparticle has a uniaxial magnetic anisotropy and size corresponding to the single-domain state. The energy of nanoparticle i can be written as a sum of the Zeeman energy in the applied magnetic field H, the energy of the dipole–dipole interaction, the anisotropy energy, and the stray-field energy, as follows:

$$W(\mathbf{m}_i) = -\mathbf{m}_i\mathbf{H} + \sum_n W_d(\mathbf{m}_i, \mathbf{m}_n) + W_a(\mathbf{m}_i) + W_s(\mathbf{m}_i). \quad (1)$$

Here, the applied magnetic field is the total of static field and high-frequency alternating field. The energy of the dipole–dipole interaction is described as follows:

$$W_d(\mathbf{m}_i) = \sum_{n\neq i}\left(\frac{\mathbf{m}_i\mathbf{m}_n r_{in}^2 - 3(\mathbf{m}_i\mathbf{r}_{in})(\mathbf{m}_n\mathbf{r}_{in})}{r_{in}^5}\right), \quad (2)$$

where $\mathbf{r}_{in}$ and r_{in} are the radius-vector and the distance between ith and nth dipoles, respectively. The energy of the uniaxial anisotropy is described as follows:

$$W_a = -\frac{K_u(\mathbf{m}_i\mathbf{n})^2}{2}, \quad (3)$$

where K_u and $\mathbf{n}$ are the constant of the uniaxial anisotropy and the unit vector along the easy axis of magnetization, respectively. The stray-field energy is determined as follows:

$$W_s(\mathbf{m}_i) = \frac{\mathbf{m}_i\hat{\mathbf{N}}\mathbf{m}_i}{2V}, \quad (4)$$

where $\hat{\mathbf{N}}$ is the tensor of demagnetization coefficients; for the case of a spherical particle, its components are $N_x = N_y = N_z = 4\pi/3$; V is the volume of the particle.

Dynamics of the moments of the dipole lattice is described by the Landau–Lifshitz equation with a relaxation term in the Gilbert form [18]:

$$\frac{\partial\mathbf{m}_i}{\partial t} = -\gamma\mathbf{m}_i\times\mathbf{H}_i^{\text{eff}} - \frac{\alpha}{m_i}\,\mathbf{m}_i\times\frac{\partial\mathbf{m}_i}{\partial t}, \quad (5)$$

where γ is the gyromagnetic ratio and α is the dissipation parameter equal for all particles. The effective magnetic field, which is induced at the position of the ith dipole by other dipoles and taking the applied field H into account (1), is described as follows:

$$\mathbf{H}_i^{\text{ef}} = -\frac{\partial W_i}{\partial\mathbf{m}_i} = \mathbf{H} + K_u\mathbf{n}(\mathbf{m}_i\mathbf{n}) - \frac{\hat{\mathbf{N}}\mathbf{m}_i}{V} + \sum_{n\neq i}\frac{3(\mathbf{m}_n\mathbf{r}_{in})\,\mathbf{r}_{in} - \mathbf{m}_n r_{in}^2}{r_{in}^5}. \quad (6)$$

Let us introduce the following dimensionless parameters: $\boldsymbol{\mu}_i = \mathbf{m}_i/m$. $\mathbf{e}_{in} = \mathbf{r}_{in}/r_{in}$, $\tau = \left(m\gamma/R^3\right)t$, where R is the nanoparticle radius; $l_{in} = r_{in}/R$. With dimensionless parameters Eq. (5) becomes as follows:

$$\frac{\partial\boldsymbol{\mu}_i}{\partial\tau} = -\boldsymbol{\mu}_i\times\mathbf{h}_i^{\text{eff}} - \alpha\,\boldsymbol{\mu}_i\times\frac{\partial\boldsymbol{\mu}_i}{\partial\tau}, \quad (7)$$

where

$$\mathbf{h}_i^{\text{eff}} = \mathbf{h} + k_u\mathbf{n}(\boldsymbol{\mu}_i\mathbf{n}) - \boldsymbol{\mu}_i + \sum_{n\neq i}\left[\frac{3(\boldsymbol{\mu}_n\mathbf{e}_{in})\,\mathbf{e}_{in} - \boldsymbol{\mu}_n}{l_{in}^3}\right].$$

In this case, the dimensionless applied field and uniaxial-anisotropy constant are determined by the following formulas: $\mathbf{h} = \mathbf{H}\,R^3/m$, $k_u = K_uR^3$.

Let us estimate dimensional quantities for the case of the dipole lattice of nanoparticles which are composed of N iron atoms. Thus, for one of the stable configurations of spherical nanoparticles $N = 561$ [19], the magnetic moment, $m \approx 2.2\,\mu_B N$, where μ_B is the Bohr magneton. In this case, the nanoparticle radius is $R = 1.364\times10^{-7}$ cm, $m \approx 1.145\times10^{-17}$ erg/G. Taking into account $\gamma = 1.76\times10^7\,(\text{Oe s})^{-1}$, we obtain the following numerical estimates for time $t = \left(R^3/\gamma m\right)\tau \approx 12\,\tau$ ps, magnetic field $H = \left(m/R^3\right)h \approx 4.5\,h$ kOe, and anisotropy constant $K_u = \left(1/R^3\right)k_1 \approx 4\times10^{20}k_u$ cm^{-3}.

As a result of further analysis we can replace Eq. (7) with three scalar equations. Thus, the x-component of $\partial\boldsymbol{\mu}_i/\partial\tau$ is described as follows:

$$\left(1+\alpha^2\right)\frac{\partial\mu_{ix}}{\partial\tau} = \left(\mu_{iz} + \alpha\mu_{ix}\mu_{iy}\right)h_{iy}^{\text{eff}} - \left(\mu_{iy} - \alpha\mu_{iz}\mu_{ix}\right)h_{iz}^{\text{eff}} - \alpha\left(1-\mu_{ix}^2\right)h_{ix}^{\text{eff}}. \quad (8)$$

Equations for the other components have the similar form and can be obtained by their cyclic permutation.

Next, we consider the 6 × 6 lattice of nanoparticles which are either the same (uniform lattice) or compose two sublattices with equal number of nanoparticles but different magnetic moments (binary lattice). In the latter case, the particles of two different types are alternating. We chose the coordinate system so that to ensure the X axis is perpendicular to the lattice plane and two other axes are parallel to its sides. The easy axis coincides with the X axis, the anisotropy constant is required to be $k_u = 1$. The dissipation parameter is required to be $\alpha = 0.01$. An equilibrium orientations and dynamic modes of precession of the total magnetic moment of the entire lattice are determined by the numerical analysis, which is carried out with the Runge–Kutta method of the fourth order.

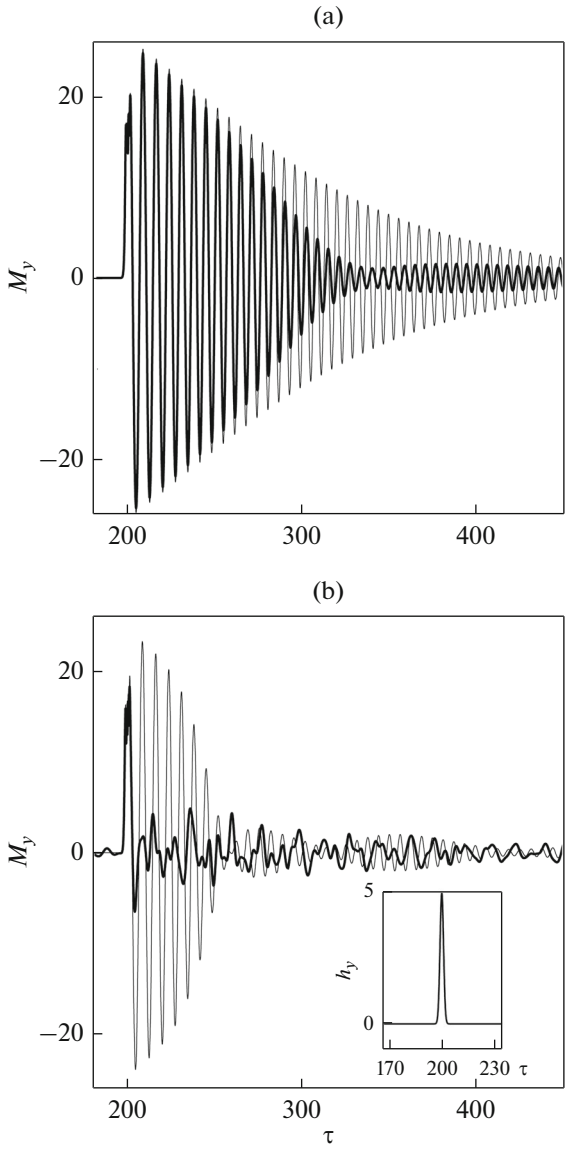

Fig. 1. Time dependence of the *y*-component of the magnetic moment of the uniform lattice under the action of pulse field with $h_0 = 5$, $\tau_i = 200$, $\tau_0 = 1$ polarized along the *Y* axis (the pulse shape is shown in the inset); the bias field $h_x = 0.1$; the dissipation parameter, $\alpha = 0.01$; the spacing between centers of the nearest nanoparticles $l_0 = 10.7$; (a) corresponds to thin and bold lines; $l_0 = 5.2$; (b) thin and bold lines.

RESPONSE OF THE SYSTEM TO MAGNETIC-FIELD PULSE

Let us consider the response of magnetic moments of the lattice under consideration to a Gaussian pulse of the applied magnetic field:

$$h(\tau) = h_0 \exp\left[-(\tau - \tau_i)^2 / 2\tau_0^2\right], \qquad (9)$$

where h_0, τ_i, and τ_0 are the peak values of field, the time shift of the pulse maximum, and the pulse duration. We require the pulse field to be linearly polarized along the *Y* axis. In the case of the uniform lattice, $|\boldsymbol{\mu}_i|$ is always equal to one. Figure 1 demonstrates the time dependence of the *y* component of the total magnetic moment $\mathbf{M} = \Sigma\boldsymbol{\mu}_i$ of the uniform lattice under the action of pulse field with the following parameters $h_0 = 5$, $\tau_i = 200$, and $\tau_0 = 1$ (the pulse shape is demonstrated in the inset). The system is also placed in the dc magnetic field aligned with the *X* axis $h_x = 0.1$. The spacing between the centers of the nearest nanoparticles is required to be $l_0 = r_0/R = 10.7$ (shown in Fig. 1a by thin and bold lines, respectively), $l_0 = 5.2$ (shown in Fig. 1b by thin and bold lines, respectively). The dipole–dipole interaction increases with decreasing distance between the nanoparticles modulates and reduces the amplitude of the system response. Further, we consider lattices with $l_0 = 10$, i.e., the low dipole–dipole interaction.

Figure 2 demonstrates the time dependence of the response duration in the case of the binary lattice under the magnetic pulse field which is polarized along the *Y* axis and has the parameters listed above. The magnetic moments of the lattice are placed in the constant magnetic field $h_x = 0.5$ directed along the *X* axis. Figure 2a corresponds to the uniform lattice with $|\boldsymbol{\mu}_i| = 1$. In the case of the binary lattice, magnetic moments of the nanoparticles of unlike sublattices are different, i.e., $|\boldsymbol{\mu}_i| = \mu_{01}, \mu_{02}$. In Fig. 2b, $\mu_{01} = 1.1$, $\mu_{02} = 0.9$, in Fig. 2c, $\mu_{01} = 1.2$, $\mu_{02} = 0.8$. We can see that the existence of two types of nanoparticles with close but different magnetic moments results in the occurrence of an additional modulating frequency, the value of which grows with increasing difference between the magnetic moments of the two dipole sublattices. In this case, a decrease in the effective field applied to the magnetic dipoles reduces the main response frequency.

DEPENDENCE OF THE RESPONSE ON PULSE DURATION

The form of trajectory and oscillation amplitude of the total magnetic moment of the lattice after the pulse application are dependent in a sophisticated way on the parameters of the lattice and pulse; in particular, the pulse duration and peak value. Figure 3 demonstrates diagrams which determine the dependence of the extremums of the *y* component of the precessing magnetic moment of the system on the pulse duration τ_0 in the time range $250 \le \tau \le 280$ after the application of the pulse field with $h_0 = 3, 5, 10, 15$ (a–d) and $\tau_i = 200$. The number of points corresponding to each

τ_0 value is approximately equal to the number of half-periods of the damped oscillations of the magnetic moment which cover the chosen time interval for a certain τ_0. The system is uniform with $|\boldsymbol{\mu}_i| = 1$, and the magnetic field $h_x = 1$ is applied along the X axis. It can be seen from the diagram that this dependence has a complicated form, i.e., the alteration of oscillations with maximal and minimal amplitudes is observed, in addition, the number of extremums in the diagram increases with the growing value of the peak pulse. So, in the case of $h_0 \le 3$, there is only one broad maximum. In the case of $h_0 = 5$, there are two apparent maximums; the first of them splits, forming a dip at the center. The further increase of h_0 enhances the number of maximums with dips. In all the cases, if the pulse duration $\tau_0 \le 0.6$, the diagram maximums are close to the limiting values $D|\boldsymbol{\mu}_i|$, where D is the number of particles in the lattice. The further increase in the pulse duration decreases maximum values in the diagram.

Figure 4 demonstrates the time dependence of $M_x(\tau)$ and the projection of the trajectory of the lattice magnetic moment on the YZ-plane after the application of the pulse with the following parameters $h_0 = 10$ $\tau_0 = 0.42, 0.52, 0.65$ (a–c). The indicated values of τ_0 approximately correspond to two maximums and a minimum of the diagram in Fig. 4c. If the pulse duration corresponds to maximums in the diagram, the response of the magnetic moments has the maximum amplitude and duration. Otherwise, if the pulse duration corresponds to a minimum in the diagram, the trajectory of the magnetic-moment oscillations rapidly approaches the equilibrium state, which limits the amplitude and results in the relatively fast damping of the oscillations. The application of the pulse with a high peak value increases the duration of the magnetic-moment precession around the polarization axis of the pulse field.

MAGNETIZATION REVERSAL BY PULSE

The application of the magnetic-field pulse can induce the partial or total magnetization reversal of the dipole lattice. First, consider the uniform lattice of nanoparticles with the magnetic moment $|\boldsymbol{\mu}_i| = 1$ which in the initial state is aligned with the positive X axis. We assume that there is no bias field $h_x = 0$. For this case, Fig. 5 demonstrates the time dependence of two components of the total magnetic moment of the lattice for the magnetization reversal by the pulse with the following parameters $h_0 = 6$, $\tau_0 = 1$ (curves *1* and *2*) and $h_0 = 7$, $\tau_0 = 0.5$ (curve *3*), $\tau_i = 200$. Curve *2* corresponds to the opposite magnetization reversal, which is characterized by the

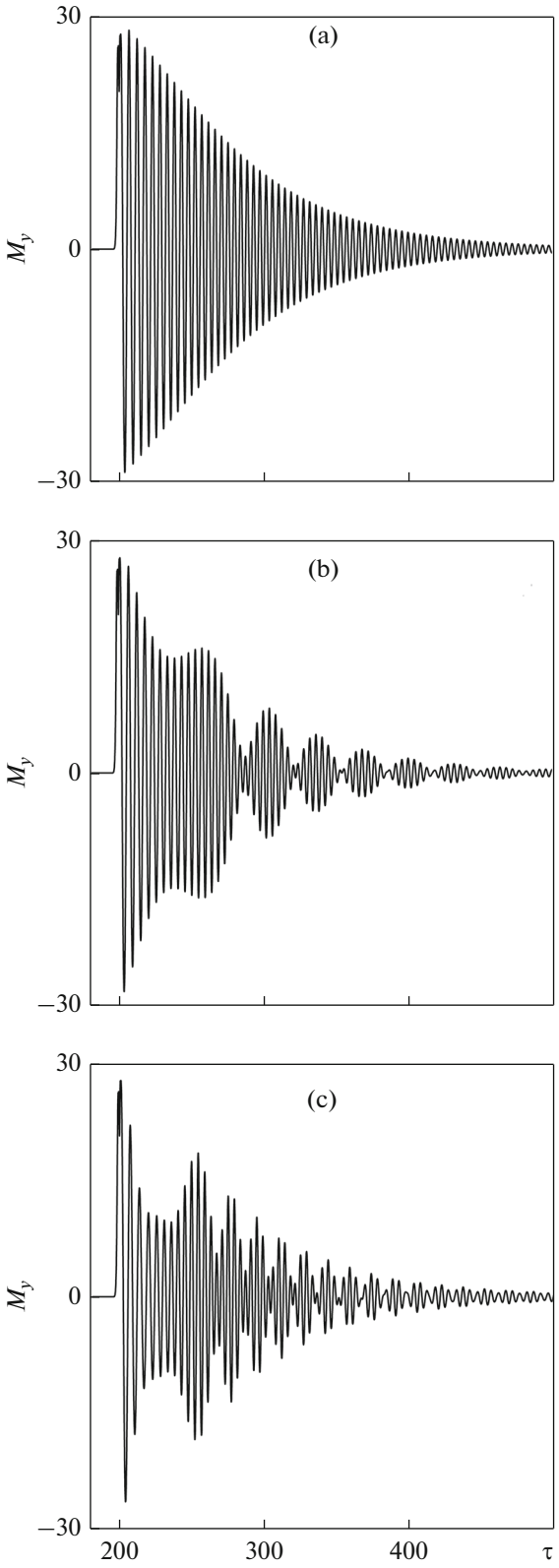

Fig. 2. Time dependence of the response of the lattice with $l_0 = 10$ to the applied pulse with $h_0 = 5$, $\tau_i = 200$, $\tau_0 = 1$, and the bias field $h_x = 0.5$; the magnetic moments: (a) $|\boldsymbol{\mu}_i| = 1$; (b) $|\boldsymbol{\mu}_i| = \mu_{01}, \mu_{02}$, $\mu_{01} = 1.1$, $\mu_{02} = 0.9$; (c) $\mu_{01} = 1.2$, $\mu_{02} = 0.8$.

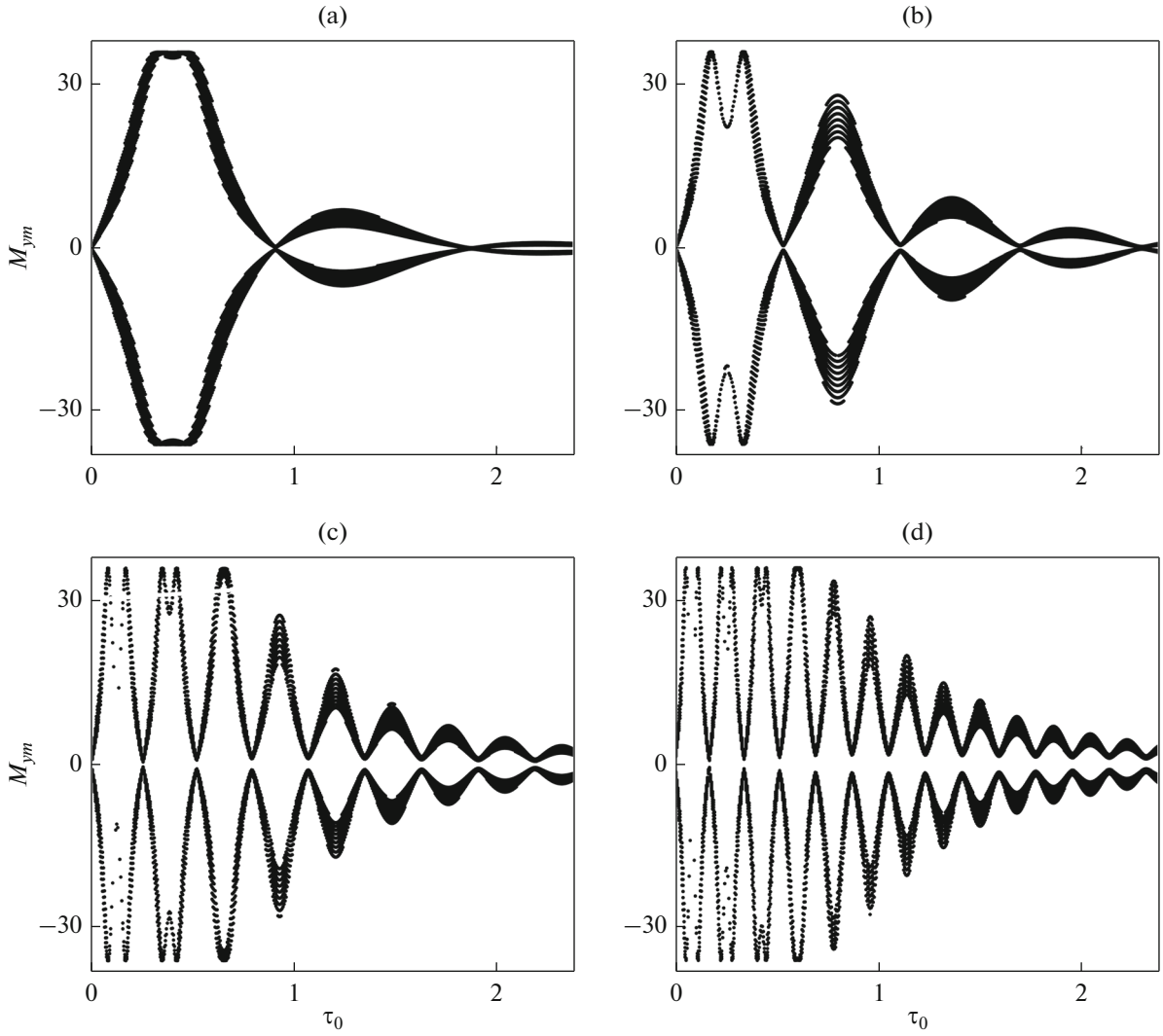

Fig. 3. Diagram of dependence of extremums of the *y*-component of the precessing magnetic moment of the system on the pulse duration with $\tau_i = 200$ and $h_0 = 3, 5, 10, 15$ (a–d) (in the range $250 \le \tau \le 280$); $|\boldsymbol{\mu}_i| = 1$, the bias field $h_x = 1$.

initial alignment of the magnetic moments of nanoparticles with the negative *X* axis. If the pulse has the following parameters $h_0 = 6$ and $\tau_0 = 0.5$, magnetization reversal is not completed, and the lattice returns to the initial configuration. To decrease the duration of magnetization-reversal pulse, the peak value of the magnetic field must be enhanced (curve *3*). In this case, with increasing h_0, the duration of the magnetization reversal decreases, and the oscillations become harmonically damped.

Further, we consider the magnetization reversal of the binary lattice which consists of alternating dipoles with $\mu_{01} = 1$ and $\mu_{02} = 2$. There is no bias field. Figure 6 demonstrates the time dependence of the *x*-component of the lattice magnetic moment under the action of three different pulses of the magnetic field with the following parameters $h_0 = 5$, $\tau_0 = 0.7$ (curves *I*), $h_0 = 5$, $\tau_0 = 1$ (curves *2*), and $h_0 = 7$, $\tau_0 = 1$ (curves *3*), $\tau_i = 200$. The magnetization reversal proceeds via four configurations (two demonstrated and those symmetrical to them with respect to the *YZ* plane) with corresponding total magnetic moments shown in Fig. 6a. Figures 6a and 6b correspond to configurations I and II, respectively. It can be seen from the figure that depending on the parameters of the pulse both the total magnetization reversal of the binary lattice and the partial magnetization reversal of each of its sublattices can occur. Magnetization reversal opposite to the ones discussed can also occur.

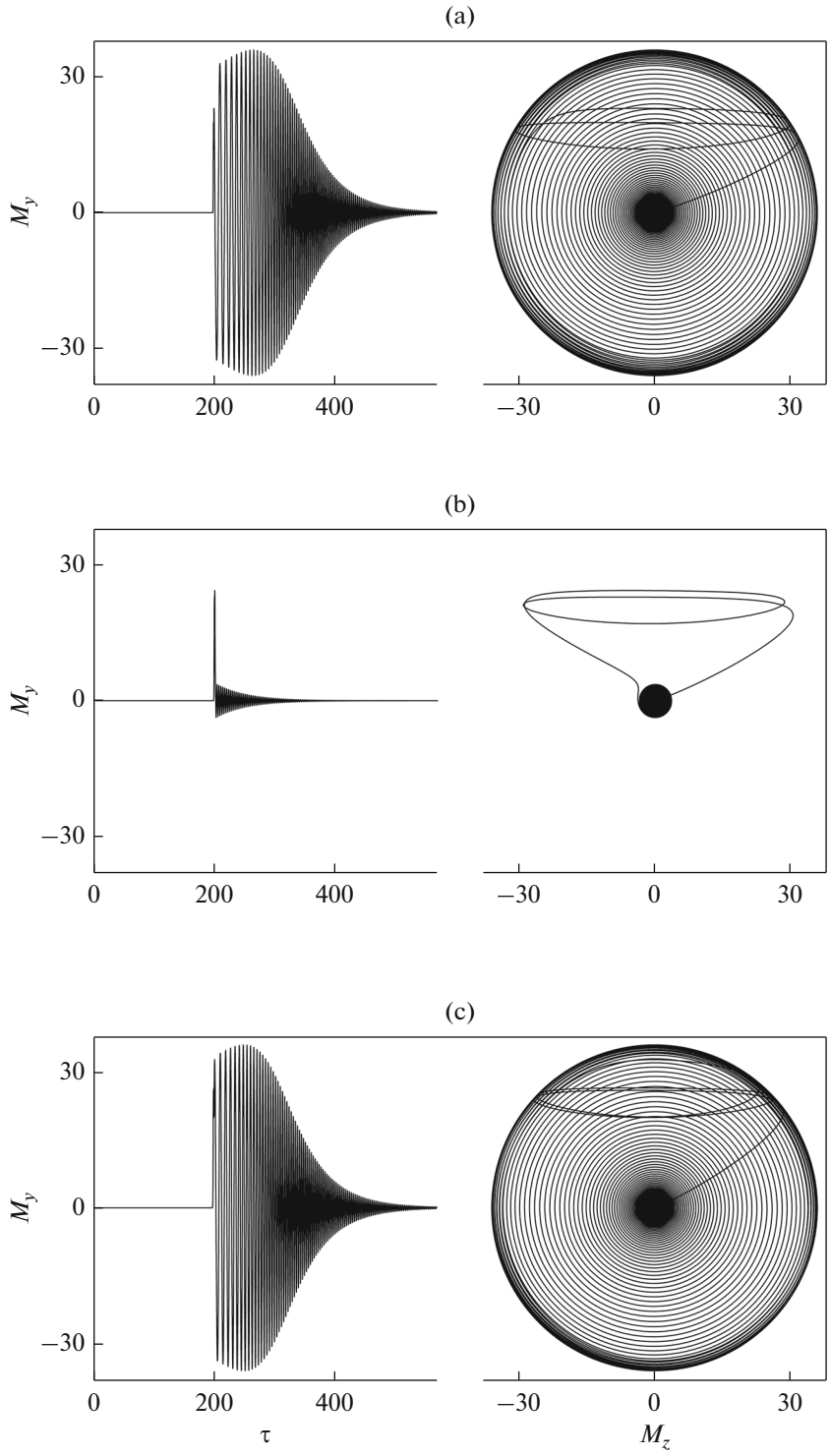

Fig. 4. Time dependence of the lattice response to the pulse with $h_0 = 10$ $\tau_0 = 0.42,\ 0.52,\ 0.65$ (a–c) and projections of trajectories of the total magnetic moment; $|\boldsymbol{\mu}_i| = 1$, $h_x = 1$.

CONCLUSIONS

The study of the dynamics of the response of magnetic nanoparticles lattices with uniaxial anisotropy to a short Gaussian pulse has demonstrated that the dipole–dipole interaction modulates the time dependence of the lattice response and decreases its amplitude. However, if the spacing between two nearest dipoles $r \geq 10R$, where R is a nanoparticle radius, in

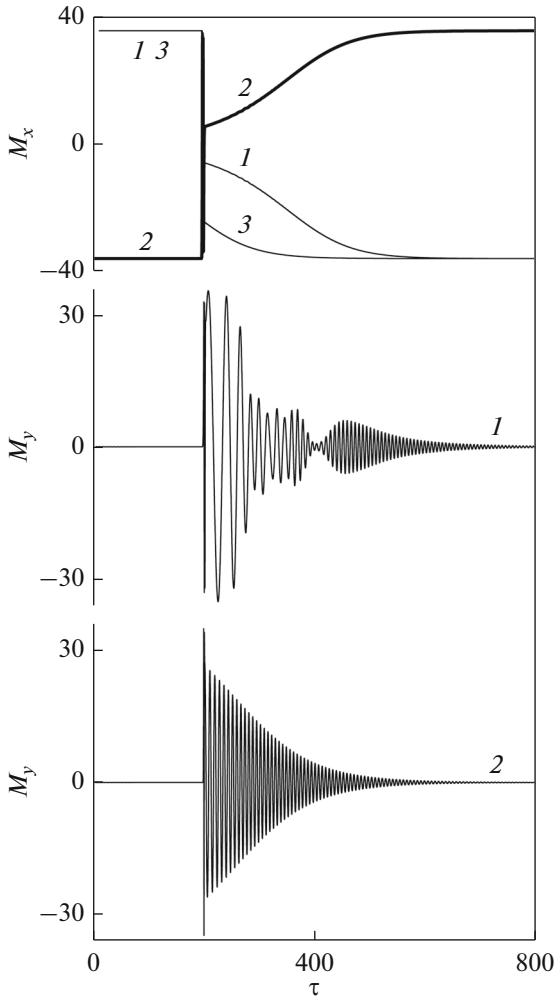

Fig. 5. Time dependence of the components of magnetic moment of the uniform lattice for the pulse of magnetization reversal with $h_0 = 6$, $\tau_0 = 1$ (curves *1*, *2*) and $h_0 = 7$, $\tau_0 = 0.5$ (curves *3*), $\tau_i = 200$; curves *1* and *3* correspond to the magnetization reversal from positive *X*-axis direction to negative one, curve *2,* the opposite process.

the majority of cases, the dipole–dipole interaction can be neglected. The presence in the lattice of two types of dipoles with close but different magnetic moments causes the occurrence of two close frequencies in the system response and the corresponding modulation of dynamics of the total magnetic moment of the lattice.

There is a strong dependence of the amplitude and duration of the magnetic-moment response to the peak value of the field and duration of its pulse. This dependence has a nonmonotonic periodic character, i.e., maximal and minimal amplitudes of the oscillations are alternating. The number of such periods grows with increasing the peak value of field pulse. In the case of pulses corresponding to the indicated minimums, the trajectory of the precessing magnetic moment rapidly approaches the dipole-equilibrium state; thus, the amplitude of the response is small. In the case of relatively short pulses, there are the regions of pulse duration which correspond to the extension of the response (by several times) of the magnetic moments.

Modes of the pulse-magnetization reversal of the lattice of nanoparticles can be controlled by the parameters of the magnetic pulse. In particular, the direct and opposite magnetization reversal of both uniform and binary lattices, as well as of each of magnetic sublattices of the latter can occur. Introduction of different types of nanoparticles into the lattice fitting the parameters of the pulse and bias field can help to achieve even more complicated modes of magnetization reversal of the parts of the system.

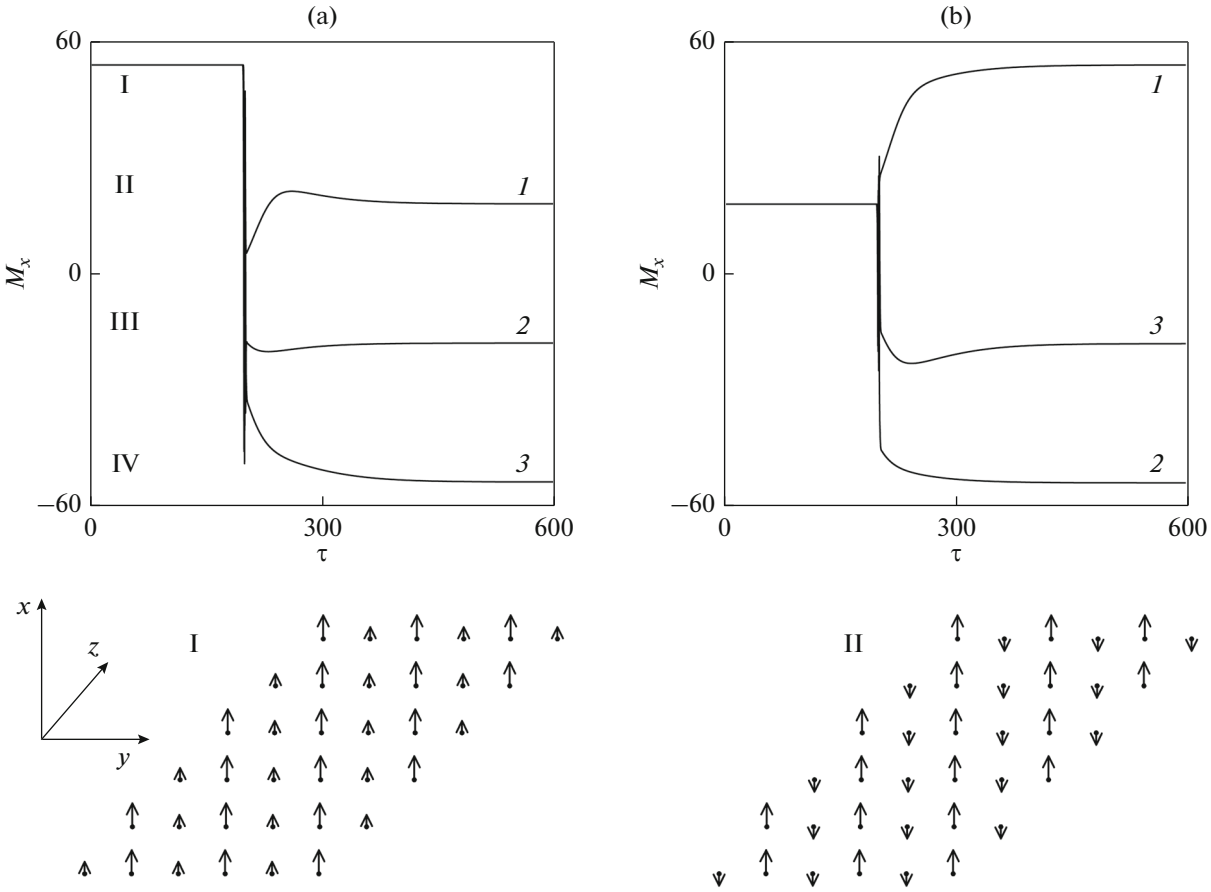

Fig. 6. Time dependence of the x-component of the magnetic moment of the lattice with $|\boldsymbol{\mu}_i| = \mu_{01}, \mu_{02}$, where $\mu_{01} = 1$, $\mu_{02} = 2$, under the action of pulse field with $h_0 = 5$, $\tau_0 = 0.7$ (curves *1*), $h_0 = 5$, $\tau_0 = 1$ (curves *2*) and $h_0 = 7$, $\tau_0 = 1$ (curves *3*), $\tau_i = 200$; there is no static field; (a) and (b) correspond to initial configurations I and II; (a) shows values of M_x for two demonstrated configurations and those symmetrical to them with respect to the YZ plane.

ACKNOWLEDGMENTS

The work was supported by the Ministry of Science and Education of the Russian Federation (State Assignment No. 3.6825.2017/BCh) and Russian Foundation for Basic Research (No. 18-42-730001/18).

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