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# Harmonically mode-locked fiber laser with fine repetition rate tuning through continuous wave injection

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## ABSTRACT

Fiber ring laser harmonically mode-locked through nonlinear polarization evolution with the pulse repetition rate up to 6.75 GHz is demonstrated. We report on a new method of fine repetition rate tuning with the use of external continuous wave injected into the ring laser cavity. With this method, we have shown experimentally that the pulse repetition rate of the harmonically mode-locked laser can be tuned over a few GHz range with the minimal possible step equal to the fundamental frequency of the ring cavity.

**Keywords:** fiber lasers, harmonic mode-locking, supermode noise suppression.

## 1. INTRODUCTION

Laser sources delivering high repetition rate pulses are of great interest for many applications in spectroscopy, microwave photonics, ranging sensing, telecommunications, etc. [1-5]. Passively mode-locked fiber lasers have become a valuable alternative to semiconductor and solid-state lasers ensuring reliability, compactness, convenient output and single-mode beam quality inherent to the laser configurations spliced in all-fiber format [6-9]. In harmonic mode-locking (HML) regime the laser emits regular pulses with much higher pulse repetition rate (PRR) equal to an integer multiple of the fundamental PRR [10-15]. Harmonic mode-locking could be implemented in the fiber laser configuration with nonlinear polarization evolution (NPE) or other kind of saturable absorbers [16-20] provided that uniform distribution of pulses along the cavity is established through their mutual repulsion governed by different processes, e.g. gain depletion and recovery [10, 14]. The pulse timing jitter in the soliton HML laser is commonly higher than that in the soliton mode-locked laser operating with the fundamental PRR [21-24]. Therefore, physical mechanisms enabling jitter reduction in HML fiber lasers are of great practical importance [25-31]. The role of background radiation as a mediator providing the equalizing interaction between pulses has been intensively discussed in this context [32-36]. Besides, the idea to handle HML through the continuous wave (CW) light injection has been raised and investigated [33] including the cases the CW light forces the laser to operate HML [34, 35]. Recent studies of the transit processes in the HML laser have confirmed an importance of the pulse interaction with the background radiation in the build-up or annihilation of soliton pulses [37]. To the best of our knowledge, the ability of the external CW light injected into the HML laser cavity to control one-by-one the birth and death of solitons has not been explored yet.

In this work, we study the effect of the CW injected into the HML laser cavity from an external CW narrow-band laser source on the HML laser operation. We demonstrate that under a proper pump power adjustment the optical injection could trigger a complex transition process in the HML laser cavity resulting in the birth or annihilation of individual solitons. The effect exhibits a strong resonant dependence on the wavelength of the injected CW light and makes no impact on other laser performance characteristics. We also explore a possible application of the observed effect for precise tuning of the HML laser PRR [38]. With perfect adjustment of the HML laser pump power, the number of solitons in the HML laser cavity could be changed simultaneously by number  $\gg 1$  [39-42]. Surprisingly, the technique based on the CW light injection enables PRR tuning with an elementary step equal to the fundamental PRR. In particular, we demonstrate the precise PRR tuning in the range of 13.46 MHz - 6.75 GHz in both positive and negative directions with the step of 13.46 MHz achieved in the HML laser configuration comprising 20.5-m-long ring cavity

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## 2. EXPERIMENT

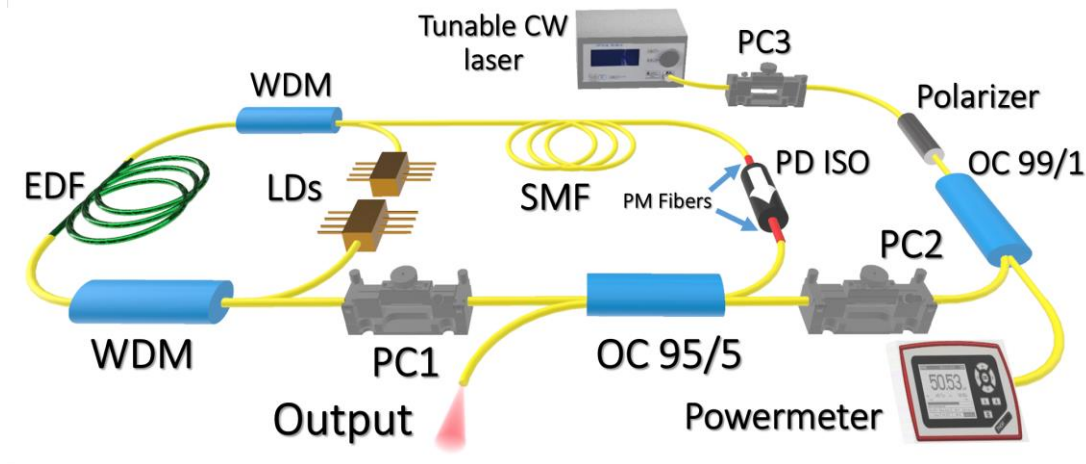


Fig. 1. Experimental setup. EDF – Er doped fiber, PC – polarization controller, PM ISO – polarization-maintaining isolator, OC – output coupler, LDs – laser pump diodes, WDMs – wavelength division multiplexors. Tunable CW laser is connected for supermode noise suppression.

The experimental configuration of an Er-doped soliton NPE mode-locked fiber ring laser is shown in Fig. 1. The laser cavity consists of two types of fibers: 0.8 m length of heavily erbium-doped fiber (EDF) with normal dispersion ( $\sim 48$  ps/nm/km) and standard single mode fiber (SMF-28) with anomalous dispersion (17 ps/nm/km). The total length of the laser cavity of 20.5 m corresponds to the fundamental PRR  $f_0 = 13.46$  MHz.

A polarization-dependent fiber isolator (PD ISO), two 980/1550 WDM couplers, in-line polarization controller (PC1) and 5% output coupler (OC) are incorporated into the cavity. The cavity is placed into a foam box to reduce the influence of the lab environment. The laser is pumped at 980 nm from two laser diodes specified for maximum power of 500mW. The central wavelength of the soliton laser can be tuned simply by adjusting PC1 that controls a linear birefringence filter formed in the cavity fiber [43-44]. The laser could be tuned to the wavelengths selected from bands between 1550 and 1590 nm specific for the built fiber configuration.

The light from the external tunable PM laser source "Yenista T100" is introduced into the HML laser cavity through the in-line polarization controller PC3, in-line polarizer, 1/99 fiber coupler (OC), polarization controller PC2, and fiber 5/95 coupler. The laser is tunable in the range of 1550-1590 nm and set to operate with the linewidth of  $\sim 100$  kHz and maximal output power of  $\sim 5$  mW. The polarization controller PC3 is used in combination with the in-line polarizer for smooth control of the CW laser power. In comparison with direct tuning of the CW laser power this method is more practical (excluding parasitic cross-effects on the laser wavelength). Note, the power and linewidth of CW light are by 2 and 6 orders of magnitude lower and narrower, respectively than those used in the experiment [35]. The polarization controller PC2 is used to control the polarization state of the CW radiation injected into the HML fiber cavity. The HML laser operation is monitored by an optical spectrum analyzer (HP 70950B) with resolution of 0.1 nm and radiofrequency spectrum analyzer (R&S FSP40) coupled with a 30 GHz photodetector (MACOM D-8IR). The injected CW power level is monitored by the photodetector attached to the 1% fiber tap.

Fig. 2 highlights details of the laser operation without optical injection. The laser wavelength is set to  $\lambda \sim 1562$  nm. Once the lasing threshold (30 mW) is achieved, the mode-locking operation is established. At a low pump power level ( $\sim 50$  mW) the laser operates regular pulses with the fundamental PRR  $f_0$ . In this regime only single soliton pulse circulates in the laser cavity. With an increase of the pump power the laser switches to multi-pulse regime. At this stage, an extra delicate adjustment of PC1 regularizes the generated pulses enabling HML laser operation. The laser emits regular pulses with the PRR equal to  $m$  pulses per one cavity round trip time,  $f_{rep} = mf_0$ . Fig. 2 (a) shows evolution of the laser average output power and the PRR with the total pump power (both laser diodes contribute equally). Hereinafter, PC1 setting is kept fixed making the presented data completely reproducible. The total pump power is increased up to 1W and then decreased down to  $\sim 60$  mW. With the maximal pump power of  $\sim 1$ W, the PRR gets  $\sim 6.75$  GHz. Red and blue lines highlight the soliton hysteresis effect [45]

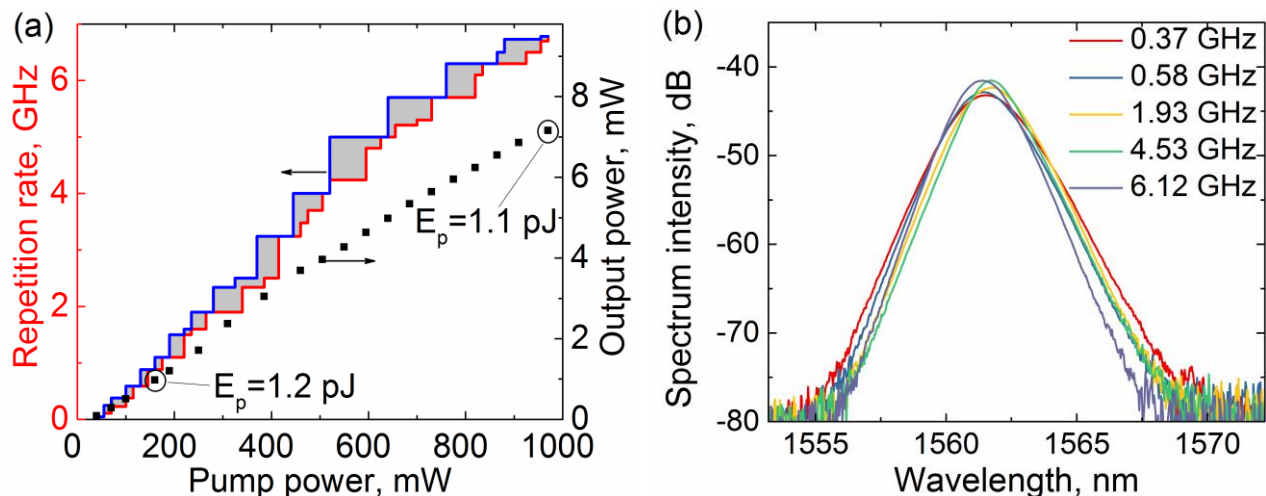


Fig. 2. (a) PRR as functions of the increasing (red line) and decreasing (blue line) pump power. The output power is shown by black squares. The laser wavelength is  $\lambda \sim 1562$  nm. In gray areas, PRR can be tuned with the step of  $f_0$ . (b) Optical laser spectrum at different PRR.

reflecting instantaneous PRR changes in cases of increasing and decreasing pump power. The positive or negative PRR jumps  $\Delta f_{rep} = \Delta m f_0$  are associated with simultaneous birth or annihilation of  $\Delta m$  solitons. The PRR points match black square points describing the laser output power in Fig.2(a). Analyzing these data, we have concluded that the laser output power is almost a linear function of the laser PRR that allows to estimate the laser energy accounted for a single soliton. It slightly changes from 1.2 pJ at 580 MHz down to 1.1 pJ at 6.5 GHz (it is low to observe Kelly sidebands). Fig. 2 (b) shows the optical HML laser spectra measured at different PRRs. All spectra are centered at 1562 nm with a FWHM ranging from 2.5 nm for the 27<sup>th</sup> HML state to 2.3 nm for the 455<sup>th</sup> HML state (transform limited pulse duration changes from  $\sim 1.02$  ps to  $\sim 1.12$  ps). The pulses at higher PRR possess narrower optical spectra.

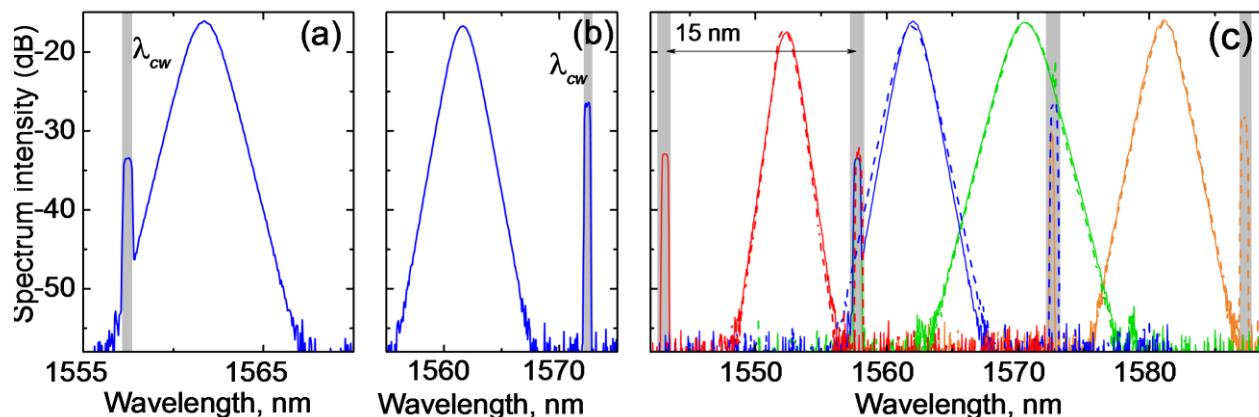


Fig. 3. Optical laser spectrum with the CW light injected at  $\lambda_{cw1}$  (a),  $\lambda_{cw2}$  (b) and measured at different positions of PC1 (c). The marked grey areas show the bands suitable for resonant injection of the CW light.

The result of CW light interaction with the HML laser radiation inside the cavity depends on the wavelength, power and polarization state of the injected light, all adjusted independently. It is due to the fiber birefringence filter formed in the HML laser cavity that exhibits a periodic spectral transmittance for polarized radiation. In operation of the HML laser based on the NPE this filter plays a crucial role. We can expect the fiber birefringence filter to produce a similar effect on the injected CW light propagating inside the HML laser cavity. For better interaction inside the HML laser cavity the polarization state of the injected CW light has to be adjusted by PC2 to the polarization state of the HML laser radiation (at least, on average). In this case, the transmission characteristics of fiber birefringent filter for the CW light are the same as for the HML laser. The use of the injected CW light at the wavelength near the filter transmission maximum allows to maximize the injected CW light power circulating inside the cavity. This intuitive analysis is in a good agreement with our experimental observations.

We have observed that the injection of external CW light into the laser cavity operating HML does not affect the laser operation itself but can change the PRR of generated pulses. To get the effect with the HML laser operating at 1562 nm (see, Fig.2) the wavelength of the injected CW laser should be selected within one of two narrow bands (typically <0.2 nm) centered at  $\lambda_{1cw} \approx 1557.5$  nm (Fig. (a)) and at  $\lambda_{2cw} \approx 1572.5$  nm (Fig. 3 (b)). These spectral bands become broader (up to ~ 1 nm) with an increase of the CW power (the PC2 responsible for the polarization state of the injected CW light inside the HML laser cavity is set once for the best effect and hereinafter kept fixed). Based on the above, we have concluded that the wavelengths  $\lambda_{1cw}$ ,  $\lambda_{2cw}$  are close to the transmittance peaks of the fiber birefringence filter [44] surrounding the HML laser operation wavelength. Beyond these bands, no effect of the injected CW light on the HML laser behavior has been observed (see, Fig.3).

Commonly, the effect of the injected external CW light on the HML laser PRR could be demonstrated as follows. The external CW laser source is set to operate at  $\lambda_{1cw}$  or  $\lambda_{2cw}$  with the maximal power level of ~5mW. The initial position of the polarization controller PC3 is set to prevent the CW light injection into the HML laser cavity. The HML laser emits the regular pulses with the PRR corresponding to the set pump power level. According to Fig.2, only a limited number of the discrete PRRs  $f_i^+ = m_i^+ f_0$  and  $f_j^- = m_j^- f_0$  are obtainable with increasing and decreasing pump powers, where  $m_i^+$  and  $m_j^-$  are the corresponding numbers of solitons in the cavity, respectively, and  $i, j = 1, 2, \dots$ . Each PRR point  $f_i^+$  or  $f_j^-$  is associated with the pump power segment  $[p_i^+, p_{i+1}^+]$  or  $[p_{j-1}^-, p_j^-]$ , where the number of pulses in the cavity  $m_i^+$  or  $m_j^-$  does not change. The discussed effect can be observed with any initial PRR, but it is more pronounced at the pump powers, where large PRR jumps occur. In order to demonstrate the effect with the initial PRR of  $f_i^+$  or  $f_j^-$  and increasing or decreasing power, we have to set the pump power near the point  $p_{i+1}^+$  within the interval  $[p_i^+, p_{i+1}^+]$  or near the point  $p_{j-1}^-$  within the interval  $[p_{j-1}^-, p_j^-]$ , respectively. Then keeping the pump power fixed, we can accurately tune PC3 to increase gradually the CW light power injected into the HML laser cavity. Fig. 4 shows the fine tuning from 97<sup>th</sup> to 112<sup>th</sup> (a) and from 112<sup>th</sup> down to 96<sup>th</sup> (b) harmonics. With an increase of the injected power, the number of pulses inside the HML laser cavity (and PRR, correspondingly) increases or decreases one-by-one, until the next stable level of  $m_{i+1}^+$  or  $m_{j-1}^-$  shown in Figs. 2(a) is achieved (in Fig.4  $m_{i+1}^+ = 112$ ,  $m_{j-1}^- = 96$ ). Importantly, this process could be stopped at any moment by reducing the injected power down to zero. In this case, the HML laser continues to generate pulses with the PRR corresponding to the last transition. Note, it distinguishes the observed effects from the (reversible and hysteresis-free) optical injection effects reported in the experiment [35] earlier. PRR tuning could be monitored with the radio-frequency (RF) spectrum analyzer. A typical RF spectrum of the HML laser consists of the main peaks and small peaks surrounding the main peaks. The main peaks are spaced by the PRR, whereas the surrounding small peaks are spaced by the fundamental PRR,  $f_0$ .

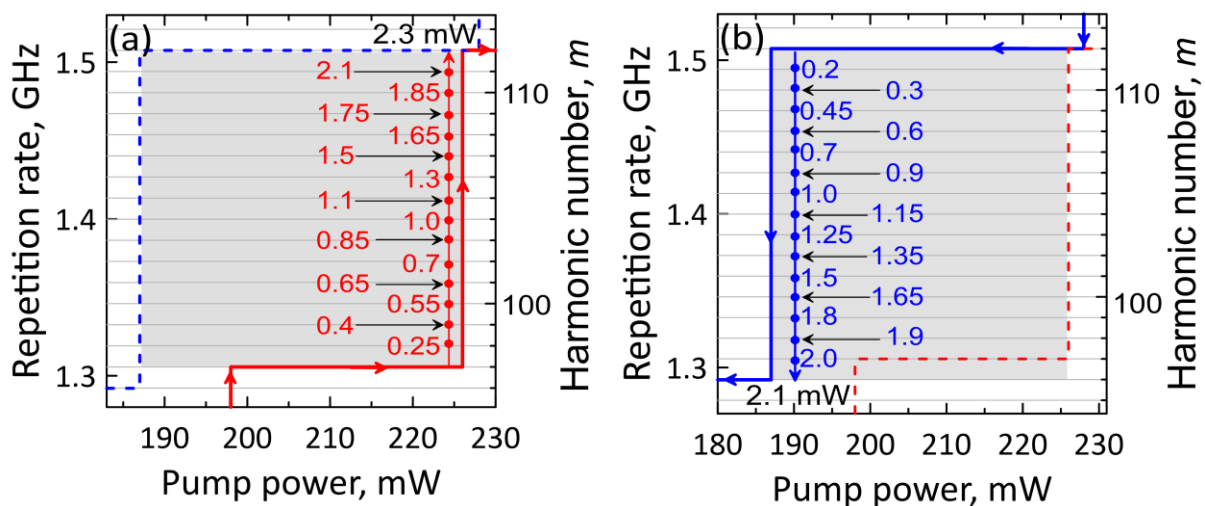


Fig. 4. Fine one-by-one tuning from 97th to 112th (a) and from 112th down to 96th (b) harmonics provided by the gradual increase of the injected CW power from 0 to 2.3 mW (a) and from 0 to 2.1 mW (b).

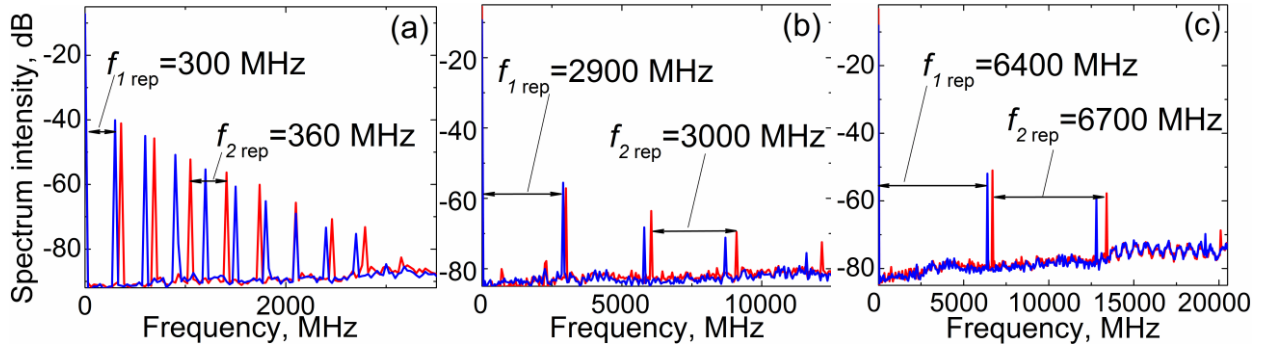


Fig. 5. Laser RF spectra before (blue lines) and after (red lines) PRR tuning at different pump power levels:  $\sim 120$  mW (a), 400 mW (b), 960 mW (c). RF spectra resolution is 30 MHz (a), 50 MHz (b, c).

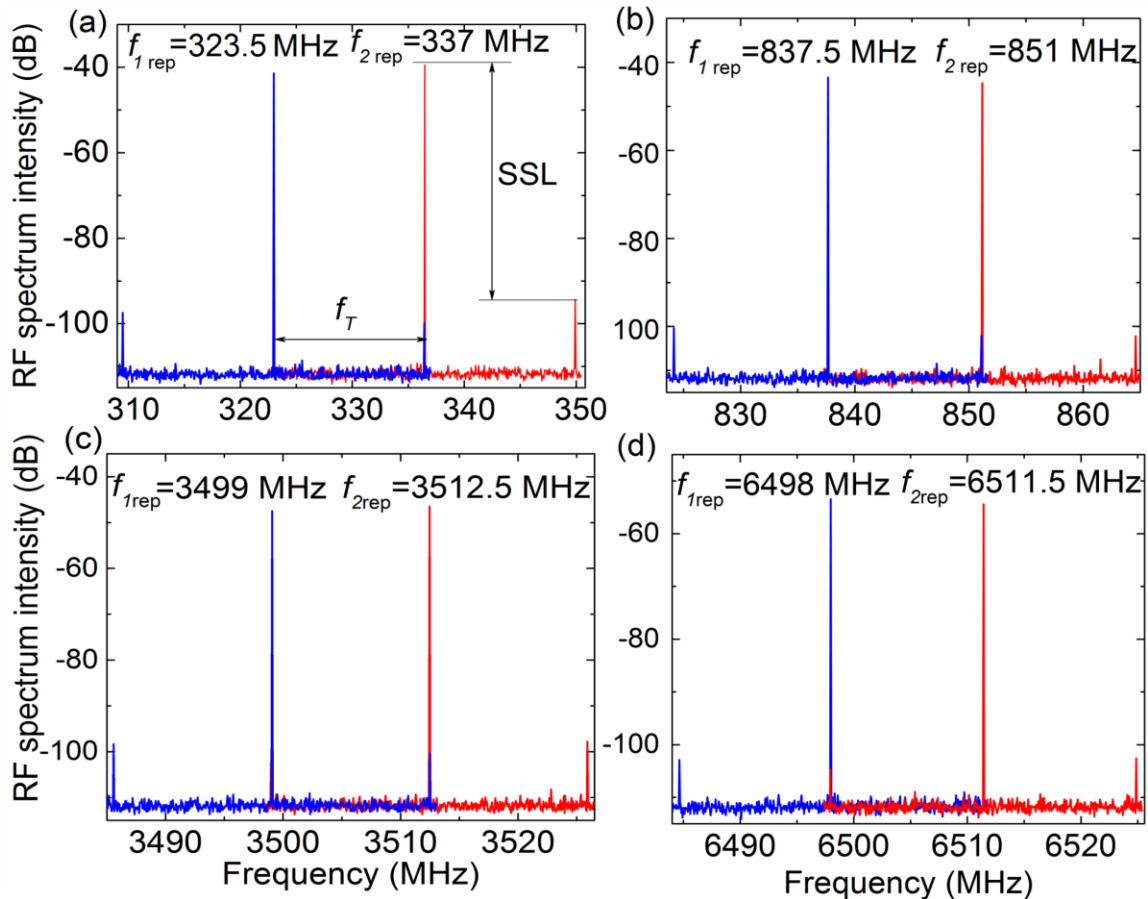


Fig. 6. Laser RF spectra before (blue lines) and after (red lines) PRR tuning with the step of  $f_0$  at different pump power levels:  $\sim 120$  mW (a), 400 mW (b), 540 mW (c), 960 mW (d). Resolution is 100 kHz.

When the CW power injected into the HML laser cavity is gradually increased, the monitored RF spectrum is perturbed and then switches to a new position corresponding to the new PRR. Fig. 5 compares the RF spectra of the HML laser radiation measured before and after PRR tuning by CW injection (at  $\lambda_{cw} \approx 1557.5$  nm, 5 mW). One can see that PRR switching does not affect the supermode noise level (SSL).

Precise control of the injected light power enables PRR switching with the elementary step equal to  $f_0$ . Fig. 6 demonstrates switching obtained with different initial PRRs. The RF spectra recorded before and after the switching are

directly compared. One can see that in all cases the optical injection with a proper power triggers a transition process in the HML laser resulting in the birth of one soliton. Importantly, the SSL remains the same after the switching. In general, it decreases with an increase of the PRR varying from  $\sim 57$  dB at 330 MHz down to  $\sim 46$  dB at 6500 MHz.

The data in Figs. 5, 6 have been obtained with the external CW source operating at  $\lambda_{1cw} \approx 1557.5$  nm. We have checked that the setting of the CW laser wavelength to  $\lambda_{2cw} \approx 1572.5$  nm gives similar results. However, in this case, the effect of the injected CW light power on the HML laser PRR is weaker, since  $\lambda_{2cw}$  is located further away from the HML laser wavelength than  $\lambda_{1cw}$ . For some initial PRRs  $f_i^\pm$ , the maximal power of the CW laser  $\sim 5$  mw is not enough to change the PRR by the maximal available value of  $\Delta m_i^\pm f_0$ . The CW light injection at the wavelength tuned to the further transmission peaks of the birefringence filter (at  $\lambda_{cw} \approx 1542.5$  nm,  $\lambda_{cw} \approx 1587.5$  nm) does not affect the HML laser operation.

We have double-checked that the described effect is independent of the HML laser wavelength. Using PC1, the laser operation with the maximal PRR around 5.5-6 GHz has been obtained at the wavelengths of  $\sim 1552$  nm, 1562 nm, 1570 nm, 1581 nm as shown in Fig.3(c). For each of these HML laser wavelengths we have found a pair of the surrounding spectral transmission peaks of the fiber birefringence filter enabling control of the HML laser PRR with the external CW laser tuned to operate at the wavelength near these peaks (see marked areas in Fig. 3(c)). The HML laser operation at different wavelengths under the CW light injection at corresponding wavelengths  $\lambda_{1cw}$  and  $\lambda_{2cw}$  is similar to that shown in Figs. 2, 3(a, b), 4-6. Features specific for the HML laser operation at different wavelengths have not been found.

### 3. DISCUSSION AND CONCLUSION

Although the detailed theoretical description of the laser operation is in progress, we believe, there is no contradiction between the reported observations and current understanding of the mechanisms underlying the birth and annihilation of solitons in the fiber laser [45]. The birth of new pulses is a natural process preventing the pulse shortening with the pump power increase. When the HML fiber laser emits regular soliton pulses with some PRR, a gradual increase of the pump power increases the energy inside the cavity leading to the broadening of the soliton pulse spectrum. The radiation filtered out by the fiber birefringence filter (and the cut-off of gain bandwidth) forms the background radiation. When further pulse shortening becomes impossible (at pump power preceding the PRR jump, see Fig.2), any further increase of the pump power injected into the soliton pulse is fully transferred to the background. According to recent experimental findings [37] the birth of an additional pulse in the HML laser cavity arises from a narrowband background component inducing narrow-band pulse that experiences strong intensity increase, while the other soliton pulses maintain their shapes. With further small power increase, the narrow-band pulse starts evolving through nonlinear pulse shaping and spectral broadening by SPM until a steady-state soliton is formed. In this understanding, we believe, that the injection of a narrow-band CW light into the HML laser cavity prevents the pulse formation from the background wave providing an alternative narrow-band seed for this process. The injection power level allows to control the soliton generation process. This explains also strong resonance properties of the effect. The injected light wavelength should match the wavelength of the narrow-band pulse generated from the background wave without seeding. The effect of soliton generation associated with the second resonant band is weaker, so it could not be implemented without the seeding due to gain competition. From the considered mechanism, PRR switching indeed does not affect the noise performance of the laser operation.

Similarly, the pulse annihilation limits further broadening of the soliton pulse with the gradual pump power decrease. According to the current understanding, this phenomenon involves instabilities at a bifurcation governed by gain and recovery dynamics [37]. Additional theoretical studies will potentially shed more insights into this evolution. We can just speculate that the injected light power triggers the pulse annihilation process providing controllable depletion to the gain. The modulation instability providing energy exchange between the injected wave and solitons can also contribute to this process due to appropriate position of the injected light wave length relative to the HML laser wavelength.

In conclusion, we have offered the method for precise PRR tuning in a soliton HML fiber laser built on NPE. The method employs a direct injection of narrow-band CW light from an external laser source into the HML laser cavity. Whereas perfect adjustment of the HML laser pump power provides only approximate PRR setting, the control of the injected CW power enables precise PRR tuning with the elementary step equal to the fundamental PRR. To demonstrate these phenomena, we have experimentally studied the soliton laser configuration comprising the 20.5-m-long ring fiber cavity and achieved precise PRR tuning in the range of 13.46 MHz - 6.75 GHz available with the step of 13.46 MHz in both (positive and negative) directions. The effect exhibits strong resonant dependence on the CW laser wavelength and

is available within two narrow-band spectral windows surrounding the HML laser wavelength. PRR switching induced by the injected CW light does not affect the laser performance characteristics. With a gradual increase of the injected CW light power the PRR changes with the elementary step one-by-one. This process could be stopped at any moment by reducing the injected power down to zero. In this case, the HML laser continues to operate pulses with the PRR corresponding to the last transition. We believe that our findings offer important insights into the transient HML laser dynamics associated with the birth and annihilation of solitons, which are crucial for the HML laser design and optimization.

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